

Week 6 – part 3 : Likelihood of a spike train



Neuronal Dynamics: Computational Neuroscience of Single Neurons

Week 6 – Noise models:

Escape noise

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↓ 6.1 Escape noise

- stochastic intensity and point process

↓ 6.2 Interspike interval distribution

- Time-dependent renewal process
- Firing probability in discrete time

6.3 Likelihood of a spike train

- generative model

6.4 Comparison of noise models

- escape noise vs. diffusive noise

6.5. Rate code vs. Temporal Code

- timing codes
- stochastic resonance

Week 6 – part 3 : Likelihood of a spike train



✓ 6.1 Escape noise

- stochastic intensity and point process

✓ 6.2 Interspike interval distribution

- Time-dependent renewal process
- Firing probability in discrete time

6.3 Likelihood of a spike train

- generative model

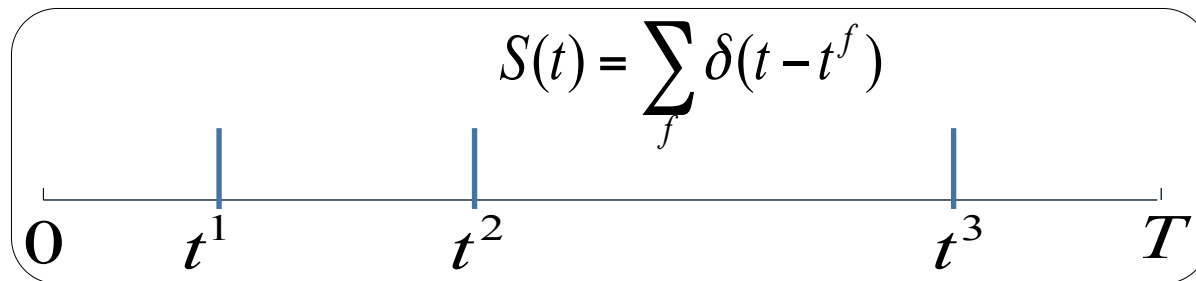
6.4 Comparison of noise models

- escape noise vs. diffusive noise

6.5. Rate code vs. Temporal Code

- timing codes
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Neuronal Dynamics – 6.3. Likelihood of a spike train



Measured spike train with spike times t^1, t^2, \dots, t^N

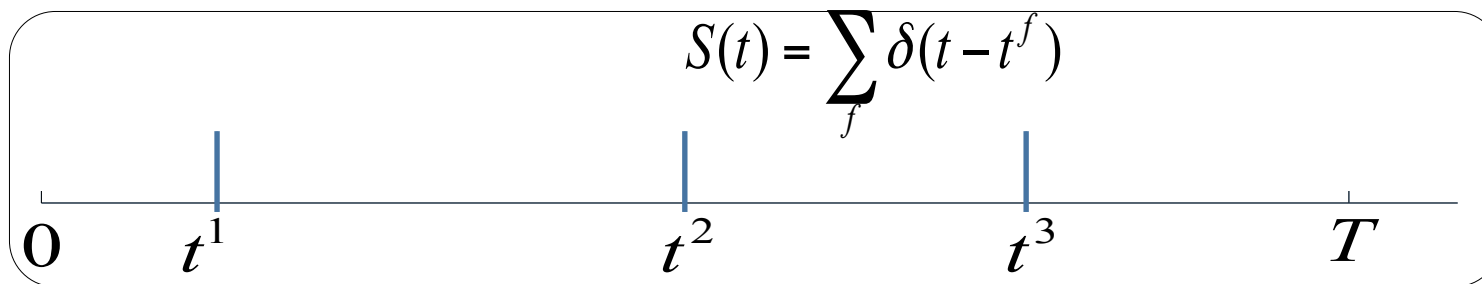
Explanation now:

Likelihood L that this spike train could have been generated by model?

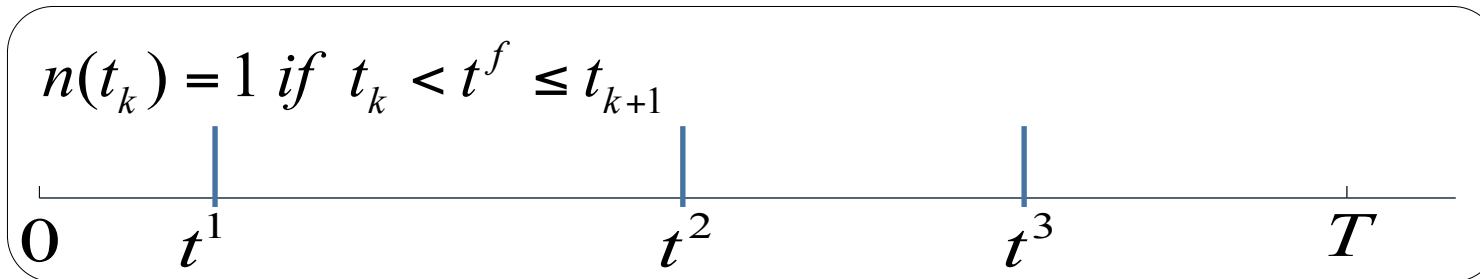
$$L(t^1, \dots, t^N) = \exp\left(-\int_0^{t^1} \rho(t') dt'\right) \rho(t^1) \cdot \exp\left(-\int_{t^1}^{t^2} \rho(t') dt'\right) \dots$$

e.g., Brillinger 1988

Neuronal Dynamics – 6.3. Likelihood of a spike train



Neuronal Dynamics – 6.3. Likelihood in discrete time



Prob. to fire in $t_k < t \leq t_{k+1}$

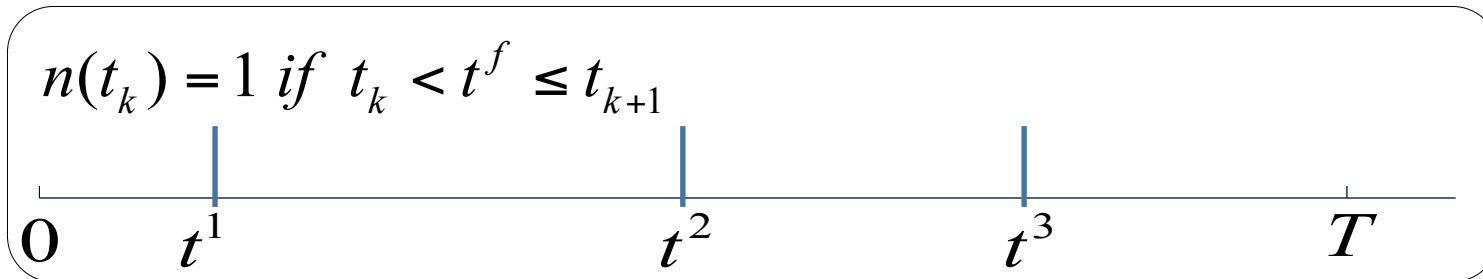
$$P_{t_k}^\Delta$$

Prob. to be silent in $t_k < t \leq t_{k+1}$

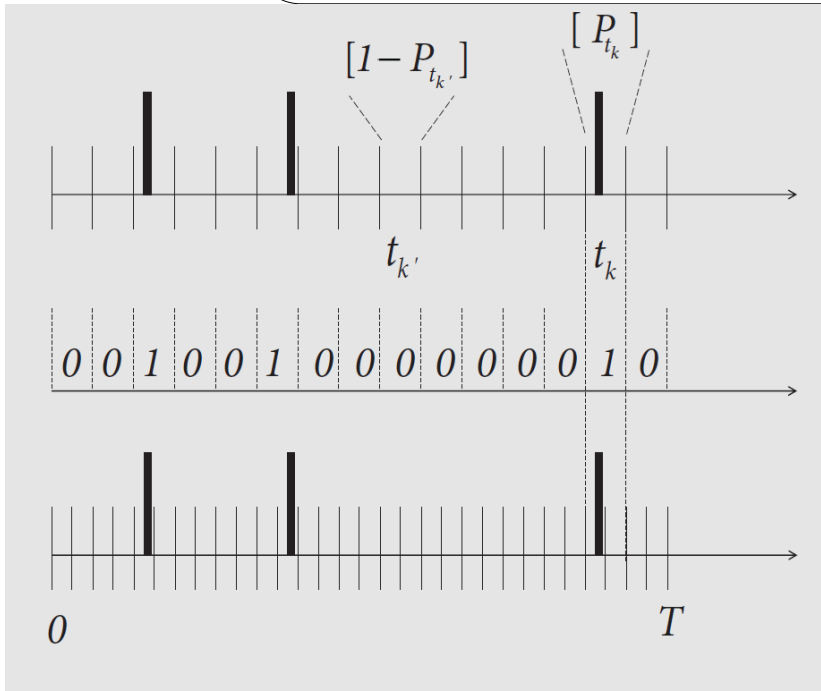
$$S^\Delta$$

how about $\Delta \rightarrow 0$??

Neuronal Dynamics – 6.3. Likelihood in discrete time

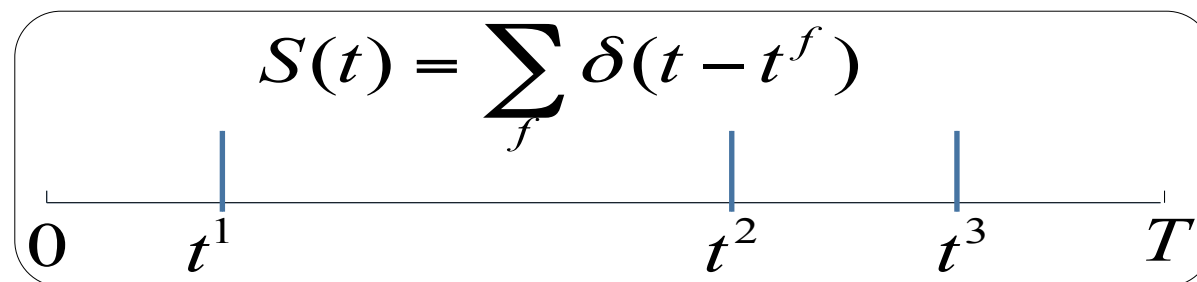
$$n(t_k) = 1 \text{ if } t_k < t^f \leq t_{k+1}$$


A horizontal timeline starting at 0 and ending at T. Three vertical blue lines mark time points t^1 , t^2 , and t^3 .



$$P_{t_k}^\Delta$$
$$\Delta \rightarrow 0$$

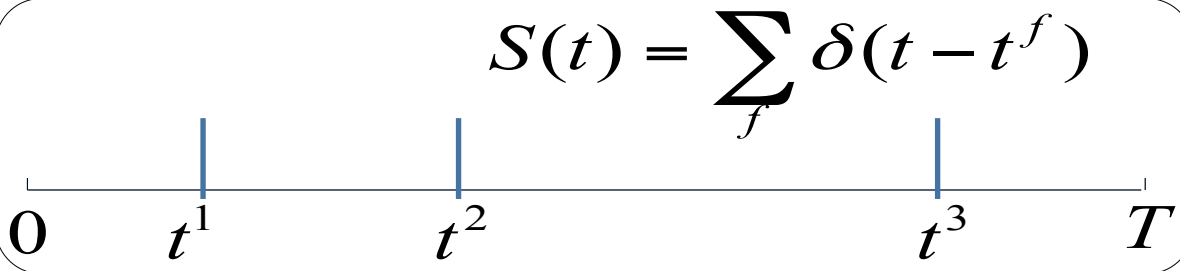
Neuronal Dynamics – 6.3. Likelihood of a spike train

$$S(t) = \sum_f \delta(t - t^f)$$


$$L(t^1, \dots, t^N) = \exp\left(-\int_0^{t^1} \rho(t') dt'\right) \rho(t^1) \cdot \exp\left(-\int_{t^1}^{t^2} \rho(t') dt'\right) \rho(t^2) \dots \exp\left(-\int_{t^N}^T \rho(t') dt'\right)$$

$$L(t^1, \dots, t^N) = \exp\left(-\int_0^T \rho(t') dt'\right) \prod_f \rho(t^f)$$

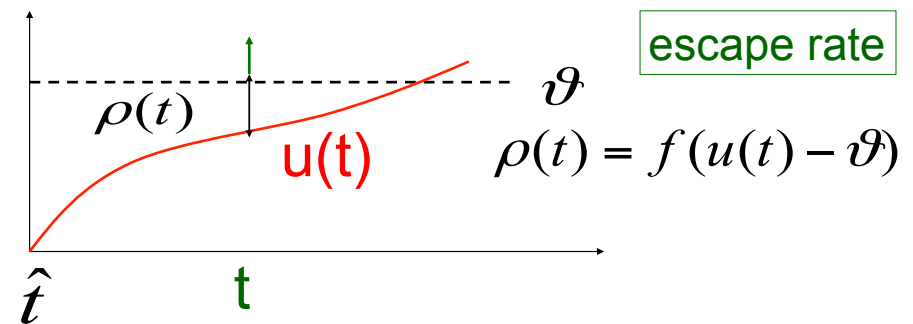
Neuronal Dynamics – 6.3. Log-likelihood of a spike train

$$S(t) = \sum_f \delta(t - t^f)$$


$$L(t^1, \dots, t^N) = \exp\left(-\int_0^T \rho(t') dt'\right) \prod_f \rho(t^f)$$

$$\log L(t^1, \dots, t^N) = -\int_0^T \rho(t') dt' + \sum_f \log \rho(t^f)$$

Neuronal Dynamics – 6.3. generative model of a spike train



generative model of spike train

- generates spikes stochastically
- calculated likelihood that an **observed** experimental spike train **could have been generated**

$$\log L(t^1, \dots, t^N) = -\int_0^T \rho(t') dt' + \sum_J \log \rho(t^J)$$

Neuronal Dynamics – Quiz 6.2. Tick all correct answers

- A leaky integrate-and-fire model with escape noise can be interpreted as a generative model of a spike train
- For a leaky integrate-and-fire model with escape noise we can (numerically) calculate the likelihood that observed experimental data could have been generated by the model
- Suppose we inject a time-dependent current into a real neuron and observe the resulting spike train. We then inject the same time-dependent current into a nonlinear integrate-and-fire model with exponential escape noise with parameter θ . For each choice of θ we can then calculate the likelihood that the model could have generated the observed spike train.