Introduction to Phasors 1:
Sinusoidal signals

If all the sources in a circuit are sinusoidal sources and our interest is in only the steady state component (because the transient component decays to approximately zero within a short time after connecting the circuit to the ac source), we can use the **phasor domain technique** to analyze the circuit.

The phasor domain technique—also known as the frequency domain technique—applies to ac circuits only, and provides a solution of only the steady state component of the total solution.

Sinusoidal signals

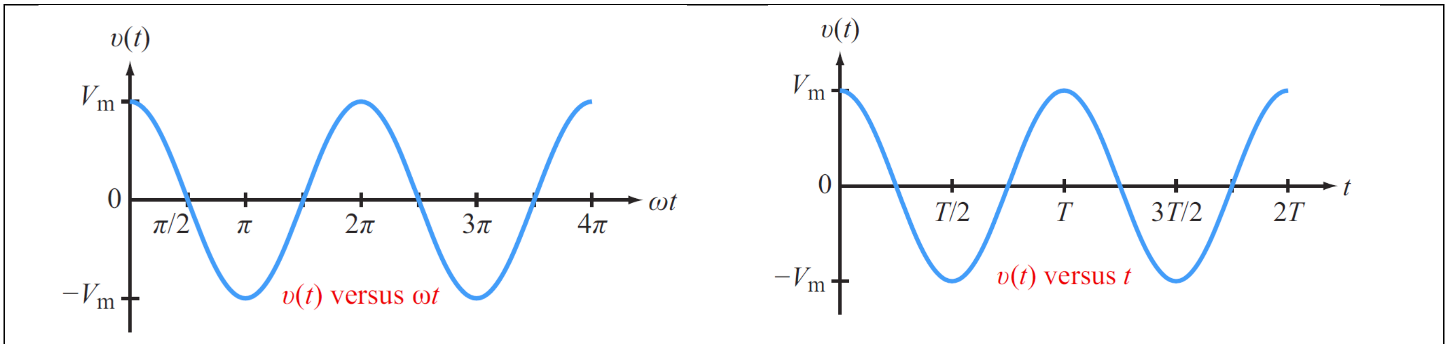
Before we can dive into the phasor technique, we need to be clear on what the mathematical expression for a sinusoidal signal tells us.

The expression

$$\mathbf{v(t) = V_m\cos(\omega t)}$$

describes a sinusoidal voltage v(t) that has an amplitude V_m and an angular frequency ω. The amplitude defines the maximum or peak value that v(t) can reach, and $-V_m$ is its lowest negative value. The argument of the cosine function, ωt, is measured either in degrees or in radians, with

$$\pi \text{ (rad)} \approx 3.1416 \text{ (rad)} = 180°$$



The function $v(t) = V_m\cos(\omega t)$ plotted as a function of (left) ωt and (right) t.

Note that the angular frequency ω is related to the oscillation frequency (or simply the frequency) f of the signal by:

$$\omega = 2\pi f \text{ (rad/s)}$$

with f measured in hertz (Hz), which is equivalent to cycles/second. A sinusoidal voltage with a frequency of 100 Hz makes 100 oscillations in 1 s, each of duration 1/100 = 0.01 s. The duration of a cycle is its period T.

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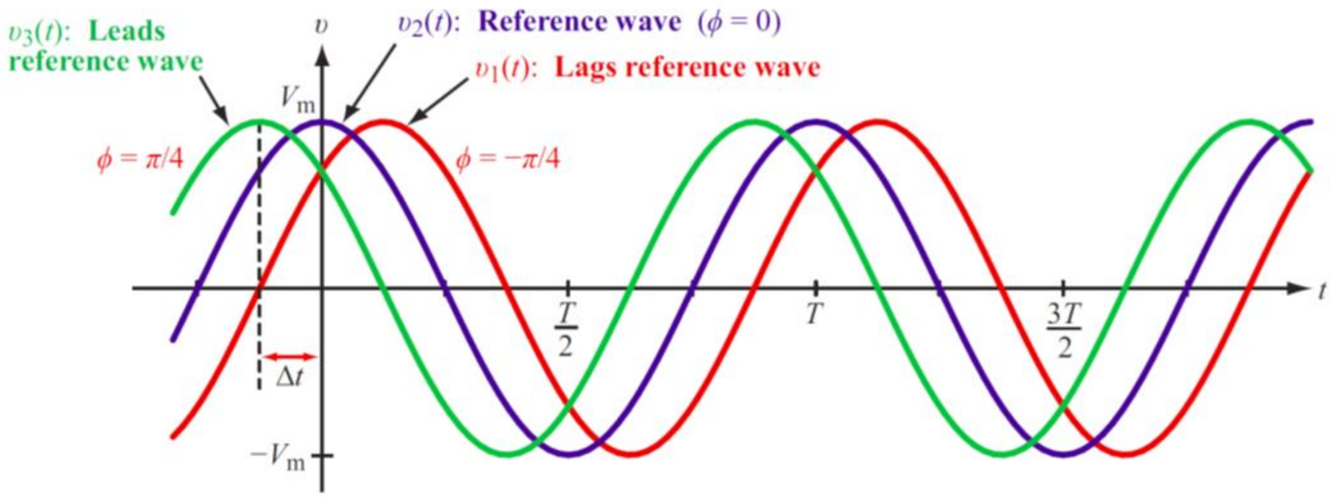
The general form of a sinusoid also includes a phase shift or phase angle, ϕ:

$$v(t) = V_m\cos(\omega t + \phi)$$

The phase shift tells us by how much the sinusoid is shifted from a reference cosine, as in the example below, which shows a reference cosine and cosines with negative and positive phase shifts.



Plots of $\upsilon(t) = V_m\cos[(2\pi t/T) + \phi]$ for three different values of ϕ.

$$v_1(t) = V_m cos\left(\frac{2\pi t}{T} - \frac{\pi}{4}\right)$$

$$v_2(t) = V_m cos\left(\frac{2\pi t}{T}\right)$$

$$v_3(t) = V_m cos\left(\frac{2\pi t}{T} + \frac{\pi}{4}\right)$$

For reference:

$$\begin{aligned}
\sin x &= \pm\cos(x \mp 90^\circ) \\
\cos x &= \pm\sin(x \pm 90^\circ) \\
\sin x &= -\sin(x \pm 180^\circ) \\
\cos x &= -\cos(x \pm 180^\circ) \\
\sin(-x) &= -\sin x \\
\cos(-x) &= \cos x
\end{aligned}$$

$$\begin{aligned}
\sin(x \pm y) &= \sin x\cos y \pm \cos x\sin y \\
\cos(x \pm y) &= \cos x\cos y \mp \sin x\sin y
\end{aligned}$$

$$\begin{aligned}
2\sin x\sin y &= \cos(x-y) - \cos(x+y) \\
2\sin x\cos y &= \sin(x+y) + \sin(x-y) \\
2\cos x\cos y &= \cos(x+y) + \cos(x-y)
\end{aligned}$$

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