Week 7 – part 2a : AdEx: Adaptive exponential integrate-and-fire

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Neuronal Dynamics:
Computational Neuroscience of Single Neurons

Week 7 – Optimizing Neuron Models For Coding and Decoding

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√ **7.1 What is a good neuron model?**
- Models and data

7.2 AdEx model
- Firing patterns and adaptation

7.3 Spike Response Model (SRM)
- Integral formulation

7.4 Generalized Linear Model
- Adding noise to the SRM

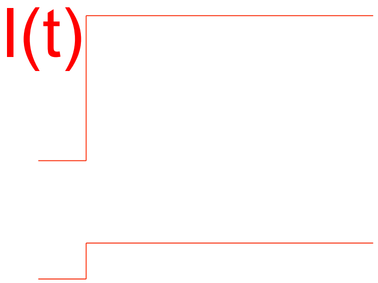
7.5 Parameter Estimation
- Quadratic and convex optimization

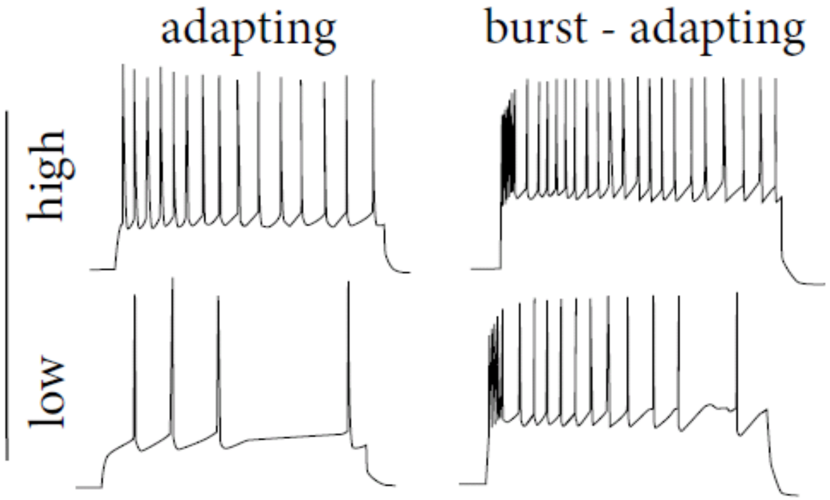
7.6. Modeling in vitro data
- how long lasts the effect of a spike?

7.7. Helping Humans

Neuronal Dynamics – 7.2 Adaptation

Step current input – neurons show adaptation





Data:
Markram et al.
(2004)

1-dimensional (nonlinear) integrate-and-fire model cannot do this!

Add adaptation variables:

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) - R \sum_k w_k$$

$$\tau_k \frac{dw_k}{dt} = a_k (u - u_{rest}) - w_k + b_k \tau_k \sum_f \delta(t - t^f)$$

Exponential I&F
+ **1 adaptation** var.
= AdEx

SPIKE AND RESET

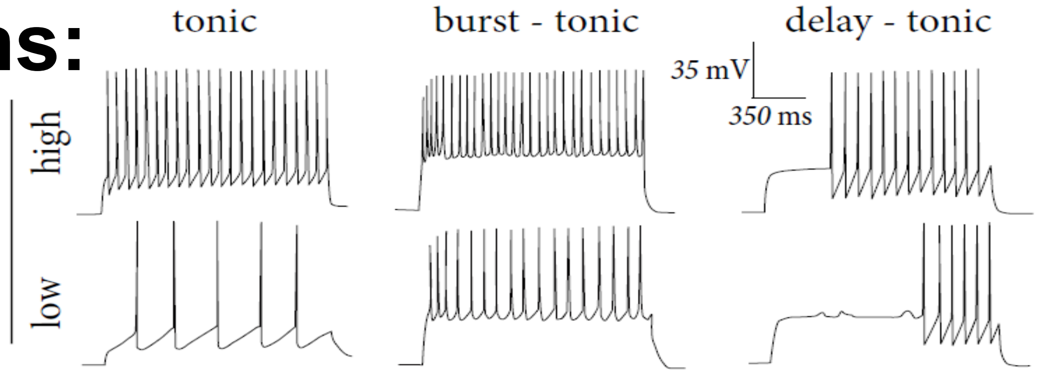
after each spike w_k jumps by an amount b_k

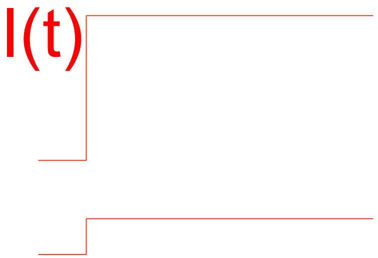
$$If \quad u = \theta_{reset} \quad then \quad reset\ to\ u = u_r$$

AdEx model, Brette&Gerstner (2005):

Firing patterns:

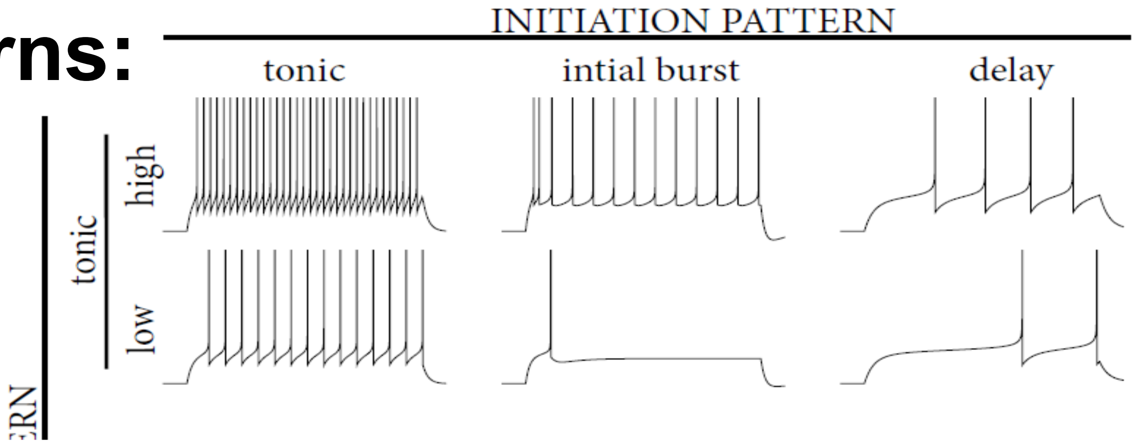
Response to
Step currents,
Exper. Data,
Markram et al.
(2004)





Firing patterns:

Response to
Step currents,
AdEx Model,
Naud&Gerstner



I(t)

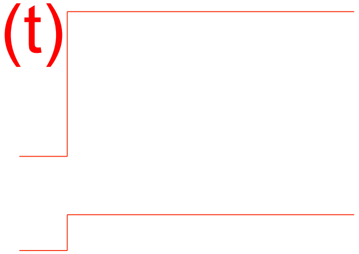


Image:
Neuronal Dynamics,
Gerstner et al.
Cambridge (2002)

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) - Rw + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\ (u - u_{rest}) - w + b\,\tau \sum_f \delta(t - t^f)$$

AdEx model

Phase plane analysis!

Can we understand the different firing patterns?

Neuronal Dynamics – 7.2. Adaptive Exponential I&F

$$\tau \frac{du}{dt} = f(u) - Rw + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\ (u - u_{rest}) - w$$

-linear + exponential
-adaptation variable

→ Various firing patterns

Neuronal Dynamics – Quiz 7.2. Nullclines of AdEx

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp(\frac{u-\vartheta}{\Delta}) - Rw + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\ (u - u_{rest}) - w$$

A - What is the qualitative shape of the w-nullcline?

[ ] constant
[ ] linear, slope a
[ ] linear, slope 1
[ ] linear + quadratic
[ ] linear + exponential

B - What is the qualitative shape of the u-nullcline?

[ ] linear, slope 1
[ ] linear, slope 1/R
[ ] linear + quadratic
[ ] linear w. slope 1/R+ exponential

Week 7 – part 2b : Firing patterns and phase plane analysis

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after each spike
u is reset to u_r

AdEx model

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right) - Rw + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\ (u - u_{rest}) - w + b\,\tau \sum_f \delta(t - t^f)$$

after each spike
w jumps by an amount ***b***

parameter $\boldsymbol{a}$ – **slope of w-nullcline**

Can we understand the different firing patterns?

AdEx model

correct

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp(\frac{u - \vartheta}{\Delta}) - Rw + RI(t)$$

correct

wrong

$$\tau_w \frac{dw}{dt} = a\ (u - u_{rest}) - w + b\ \tau\ \sum_f \delta(t - t^f)$$

Throughout this lecture 7.2b, the τ in the differential equation for w should have on both sides an index w (here correct on the left, wrong on the right)

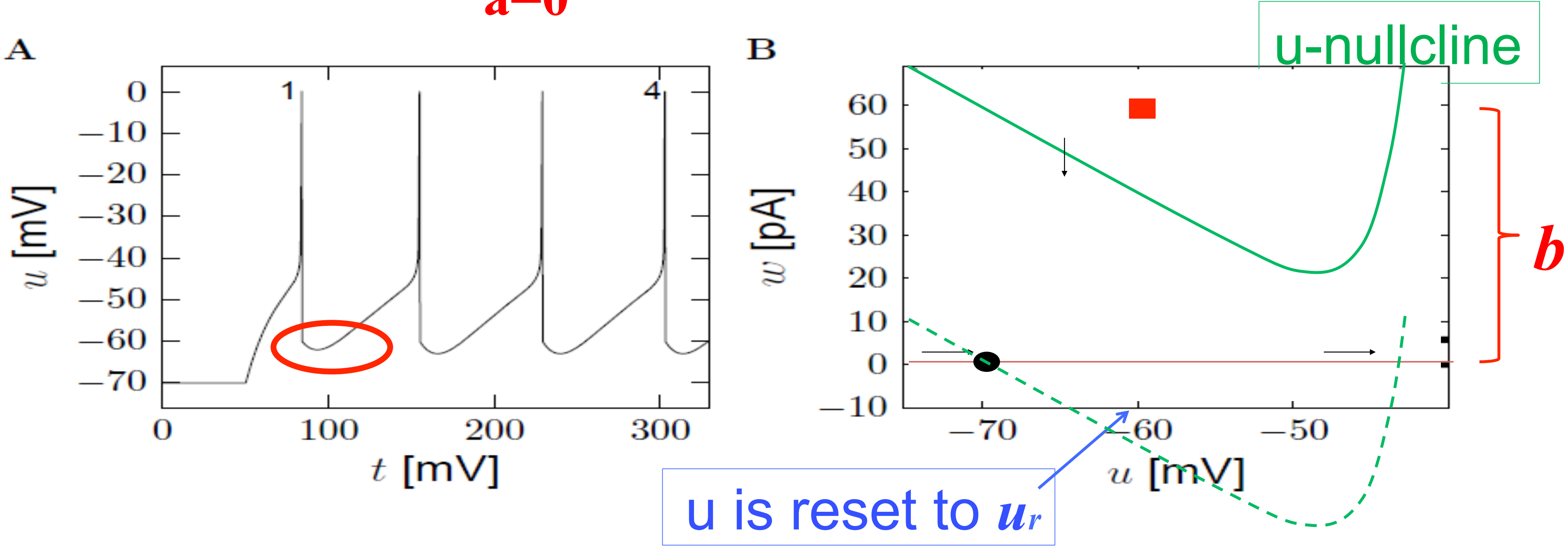
$$\tau \longrightarrow \tau_w$$

AdEx model – phase plane analysis: large *b*

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp(\frac{u-\vartheta}{\Delta}) + w + RI(t)$$

$$\tau \frac{dw}{dt} = a\ (u - u_{rest}) - w + b\,\tau \sum_f \delta(t - t^f)$$

a=0

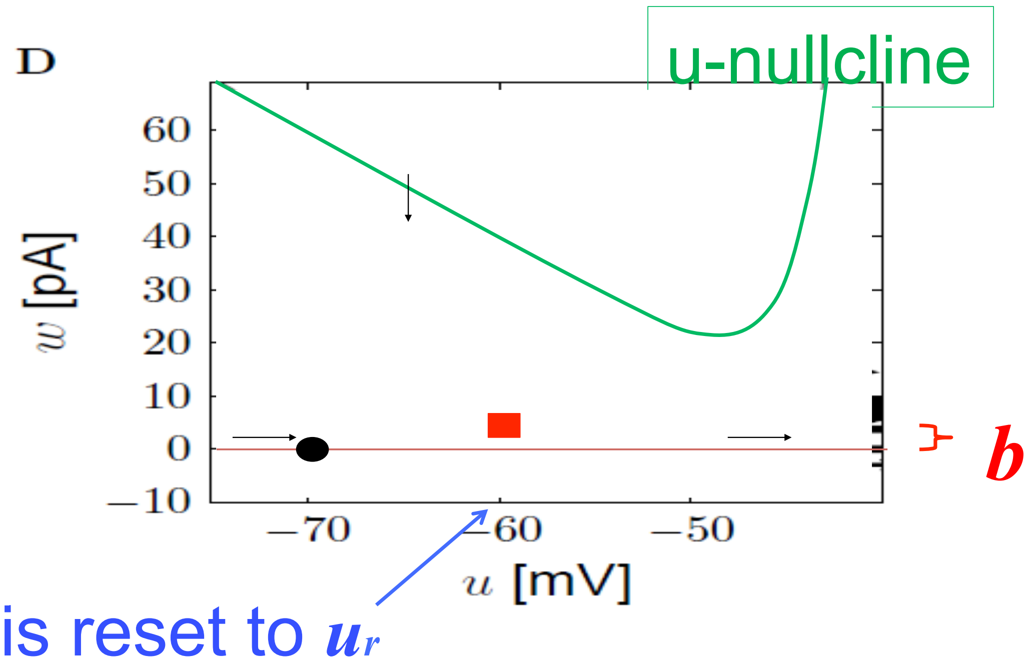


AdEx model – phase plane analysis: small *b*

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp(\frac{u-\vartheta}{\Delta}) + w + RI(t)$$

$$\tau \frac{dw}{dt} = a\ (u - u_{rest}) - w + b\,\tau \sum_f \delta(t - t^f)$$

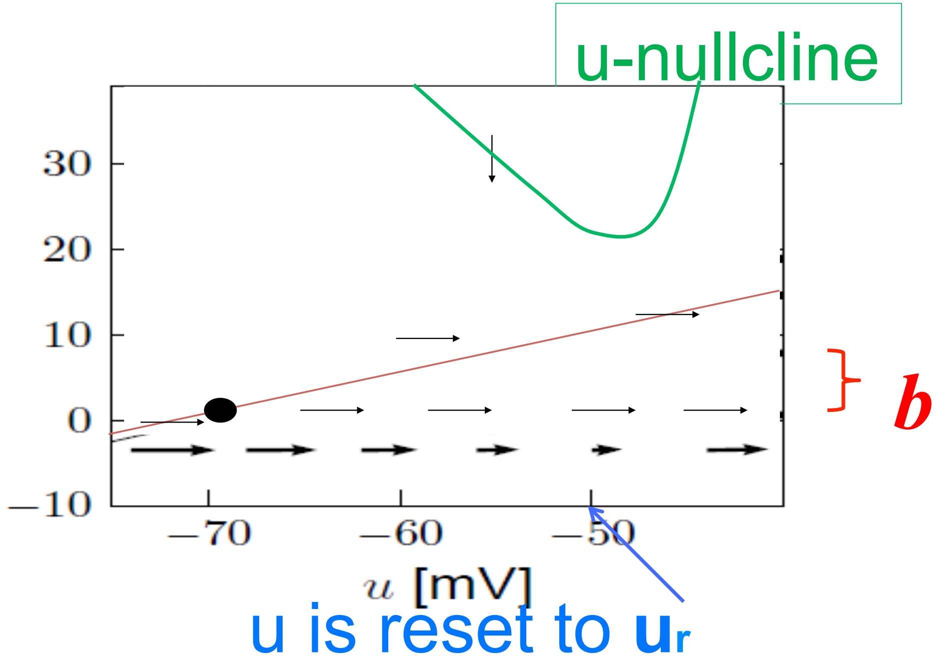
adaptation



AdEx model – phase plane analysis: a>0

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right) + w + RI(t)$$

$$\tau \frac{dw}{dt} = a\ (u - u_{rest}) - w + b\,\tau \sum_f \delta(t - t^f)$$



Neuronal Dynamics – 7.2 AdEx model and firing patterns

after each spike u is reset to $\mathbf{u_r}$

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) - Rw + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\ (u - u_{rest}) - w + b\,\tau \sum_f \delta(t - t^f)$$

after each spike
w jumps by an amount **b**

parameter **a – slope of w nullcline**

Firing patterns arise from different parameters!

See Naud et al. (2008), see also Izikhevich (2003)

Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

(1) $$\tau \frac{du}{dt} = f(u) + R\,I(t)$$

(2) $If \;\; u = \theta_{reset} \;\; then \;\; reset\, to\; u = u_r$

Best choice of ***f*** : linear + exponential

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right)$$

BUT: **Limitations – need to add**

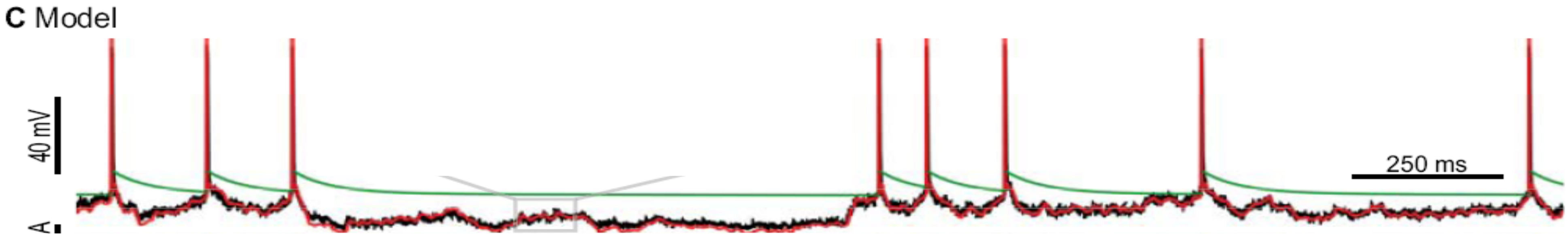
- ✓ -Adaptation on slower time scales
- ✓ -Possibility for a diversity of firing patterns
- -Increased threshold ϑ after each spike
- -Noise

Add dynamic threshold:

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp(\frac{u - \vartheta}{\Delta}) - R \sum_k w_k + RI(t)$$

Threshold increases after each spike

$$\vartheta = \theta_0 + \sum_f \theta_1 (t - t^f)$$



$$\tau \frac{du}{dt} = f(u) + R\,I(t)$$

$$If \quad u = \theta_{reset} \quad then \quad reset\, to\; u = u_r$$

add

- ✓ -Adaptation variables
- ✓ -Possibility for firing patterns
- ✓ -Dynamic threshold ϑ
- -Noise

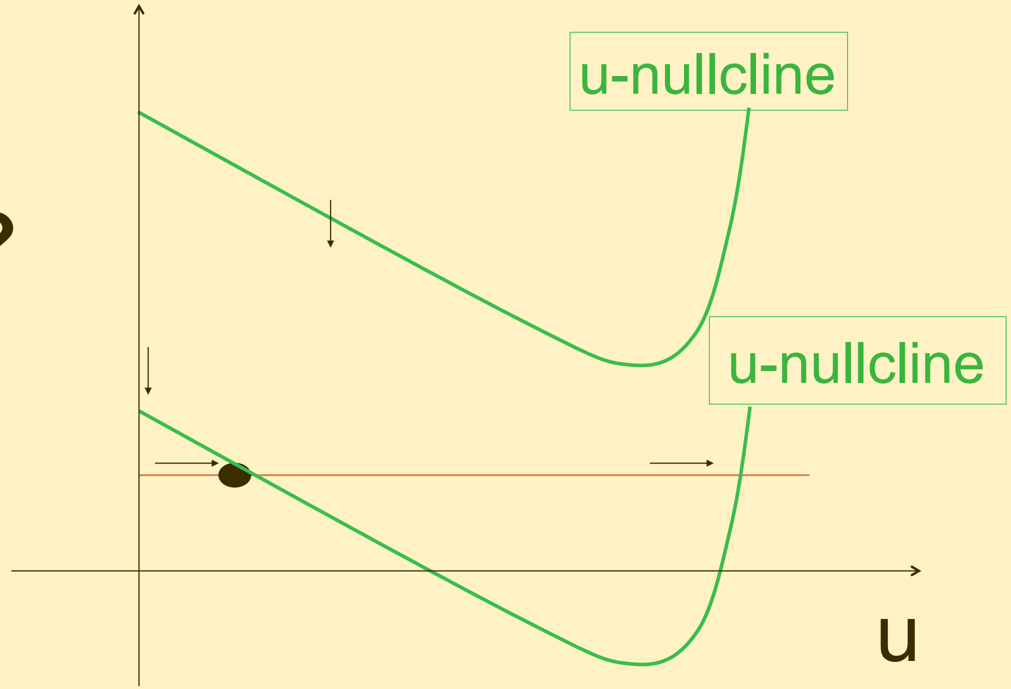
Neuronal Dynamics – Quiz 7.3. Nullclines for constant input

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp(\frac{u-\vartheta}{\Delta}) + w + RI(t)$$

$$\tau \frac{dw}{dt} = a\ (u - u_{rest}) - w + b\,\tau \sum_f \delta(t - t^f)$$

a=0

Only during reset



What happens if input switches from I=0 to I>0?

[ ] u-nullcline moves horizontally
[ ] u-nullcline moves vertically
[ ] w-nullcine moves horizontally
[ ] w-nullcine moves vertically