Week 4 – part 4b : Firing threshold in 2D models

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Neuronal Dynamics:
Computational Neuroscience of Single Neurons

Week 4 – Reducing detail:
Two-dimensional neuron models

Wulfram Gerstner

EPFL, Lausanne, Switzerland

Week 4 – part 4b : Firing threshold in 2D models

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√4.1 **From Hodgkin-Huxley to 2D**

√4.2 **Phase Plane Analysis**

√4.3 **Analysis of a 2D Neuron Model**

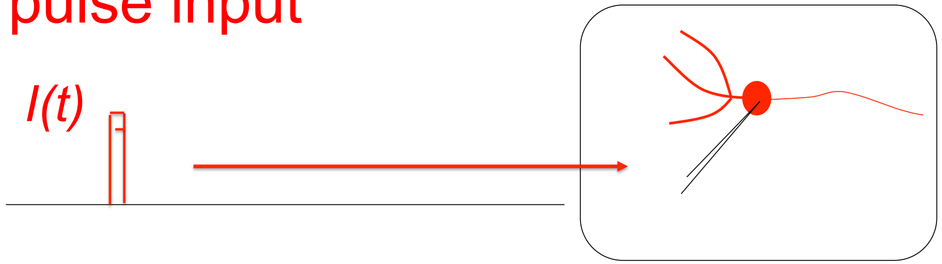
√4.4 **Type I and II Neuron Models**
- where is the firing threshold?
- separation of time scales

4.5. **Nonlinear Integrate-and-fire**
- from two to one dimension

Neuronal Dynamics – 4.4b Threshold in 2dim. Neuron Models

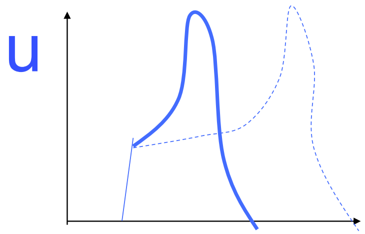
pulse input

neuron



Delayed spike

Reduced amplitude



Neuronal Dynamics – 4.4 Bifurcations, simplifications

Bifurcations in neural modeling,
Type I/II neuron models,
Canonical simplified models

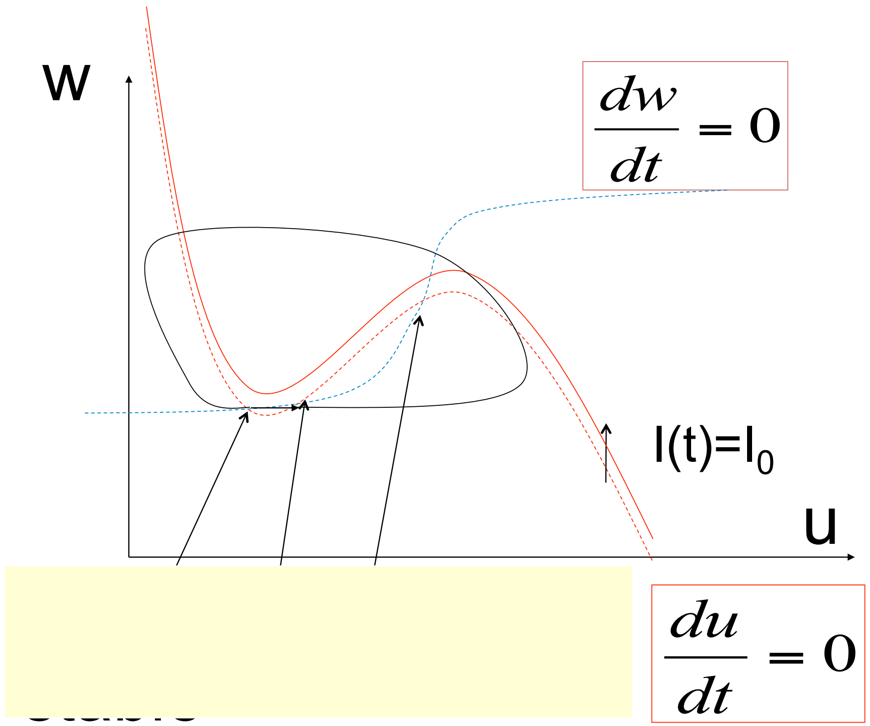
Nancy Koppell,
Bart Ermentrout,
John Rinzel,
Eugene Izhikevich
and many others

Saddle-node onto limit cycle bifurcation

stimulus

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$



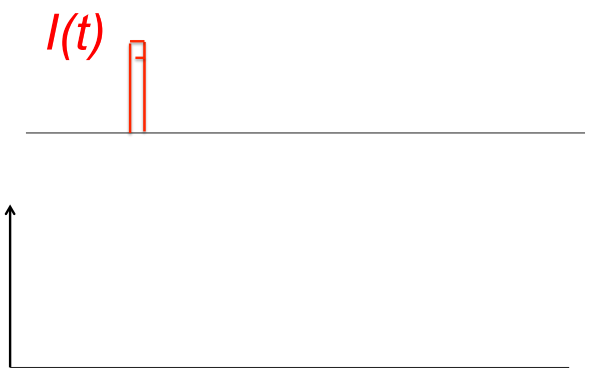
Neuronal Dynamics – 4.4b Pulse input

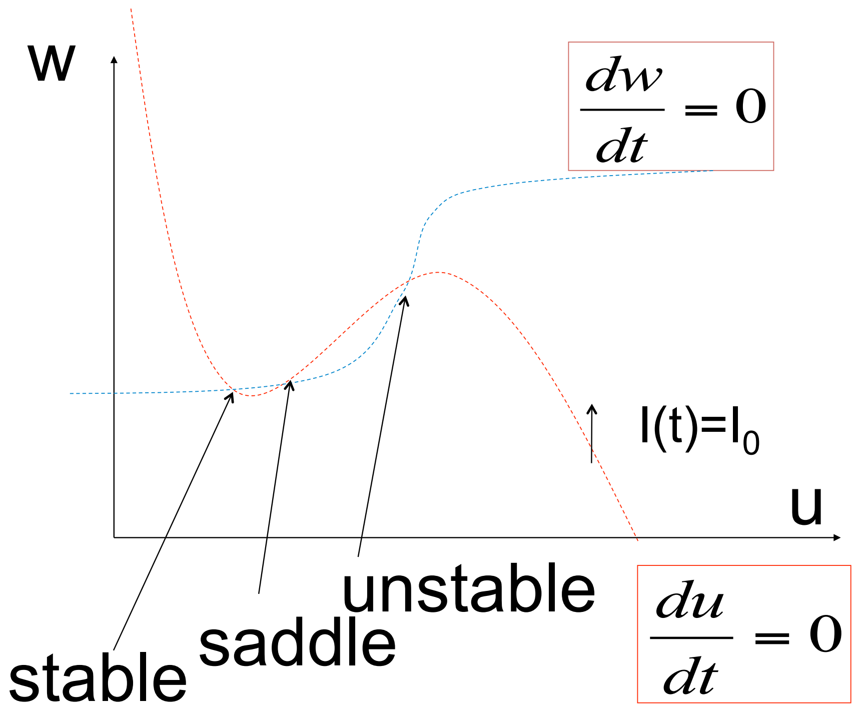
stimulus

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

pulse input





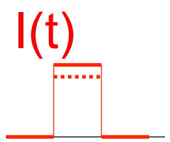
4.4b Type I model: Pulse input

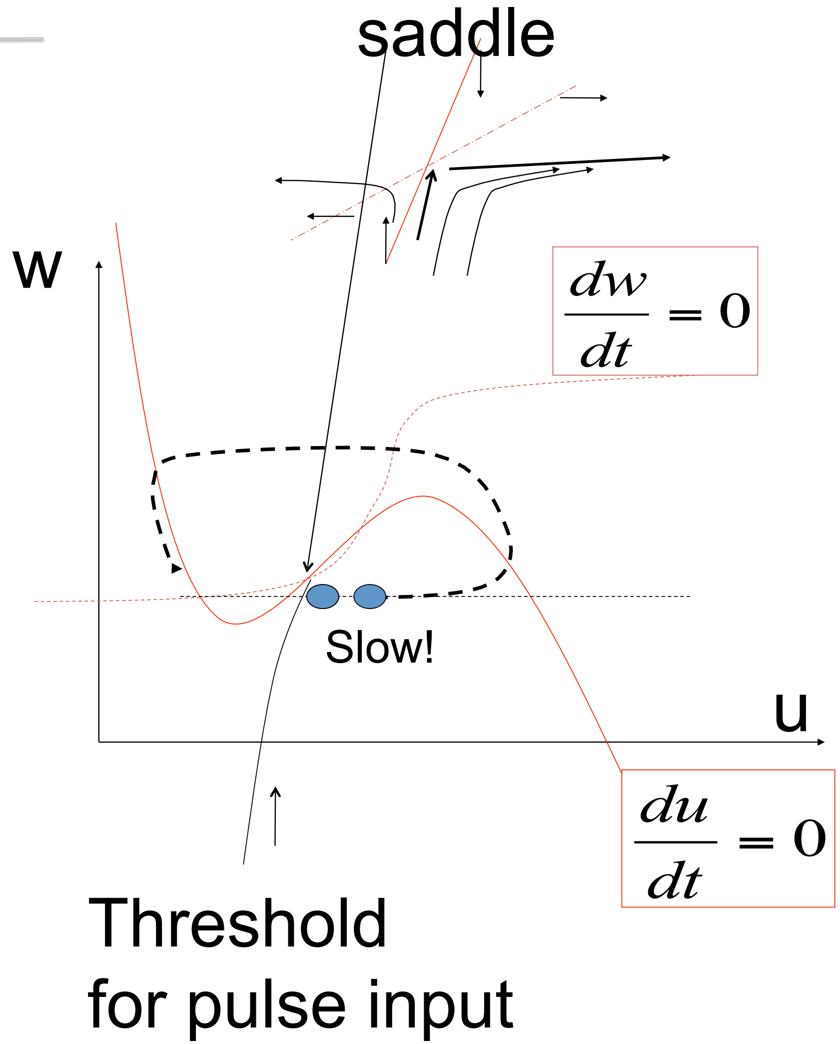
stimulus

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

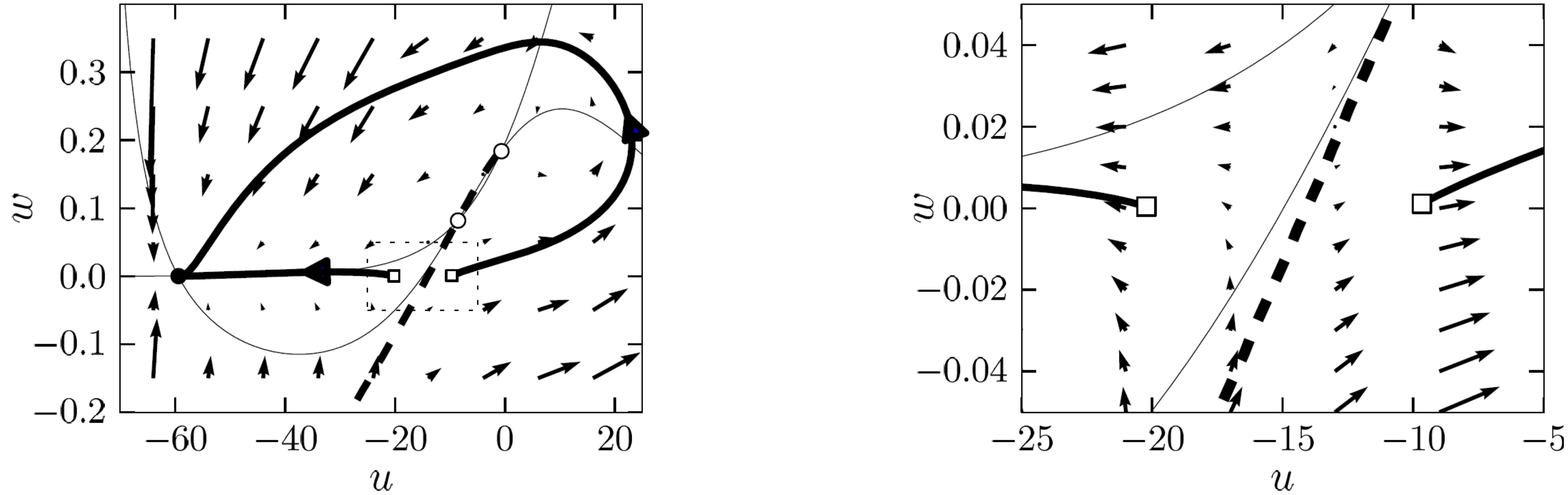
$$\tau_w \frac{dw}{dt} = G(u,w)$$

pulse input





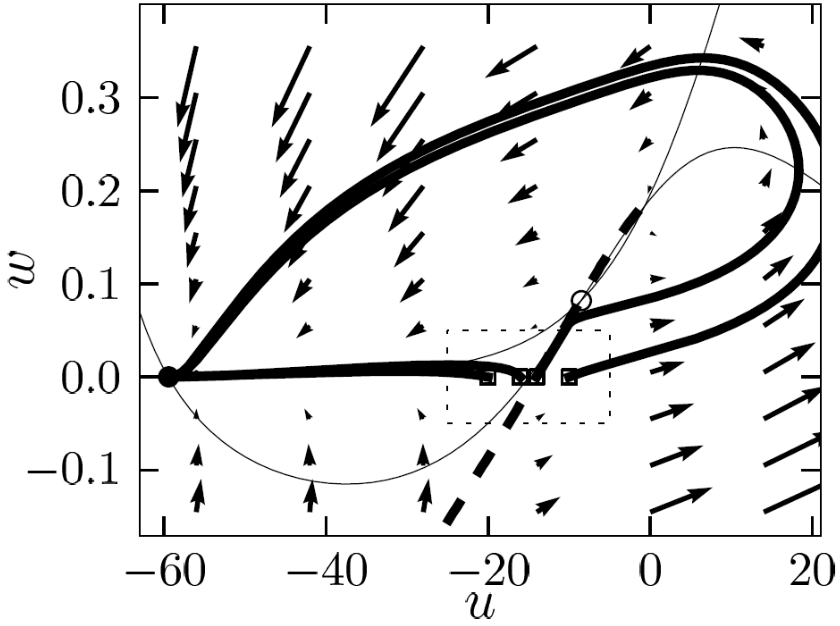
4.4b Type I model: Threshold for Pulse input

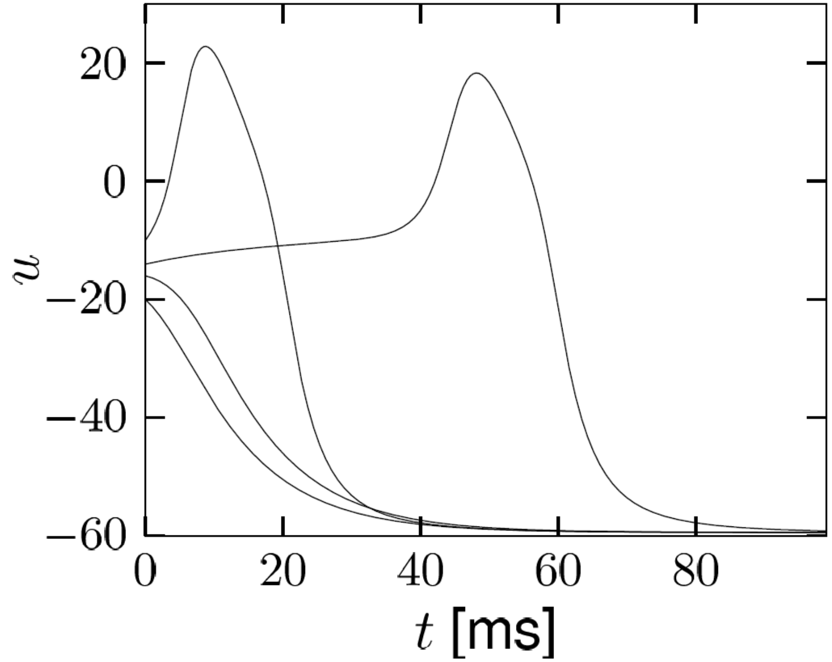


Stable manifold plays role of 'Threshold' (for pulse input)

Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

4.4b Type I model: Delayed spike initation for Pulse input

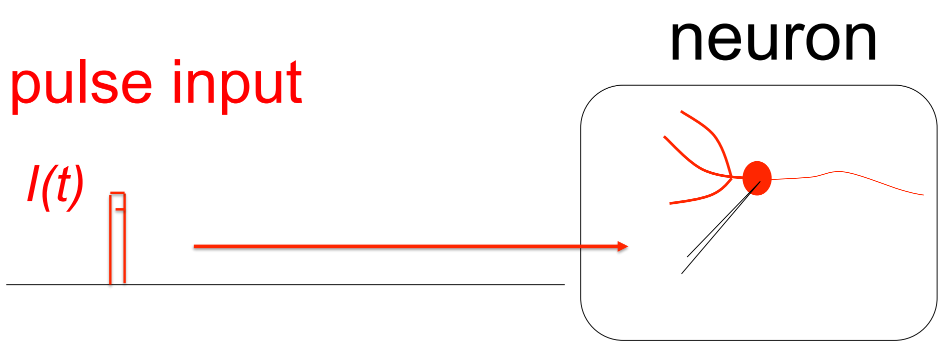




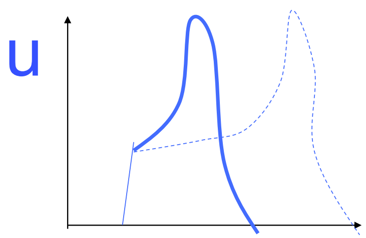
Delayed spike initiation close to 'Threshold' (for pulse input)

Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

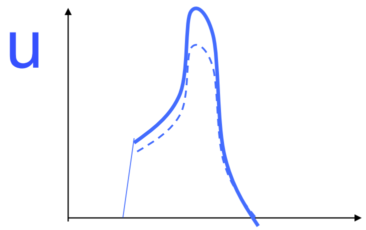
Neuronal Dynamics – 4.4b Threshold in 2dim. Neuron Models



Delayed spike



Reduced amplitude



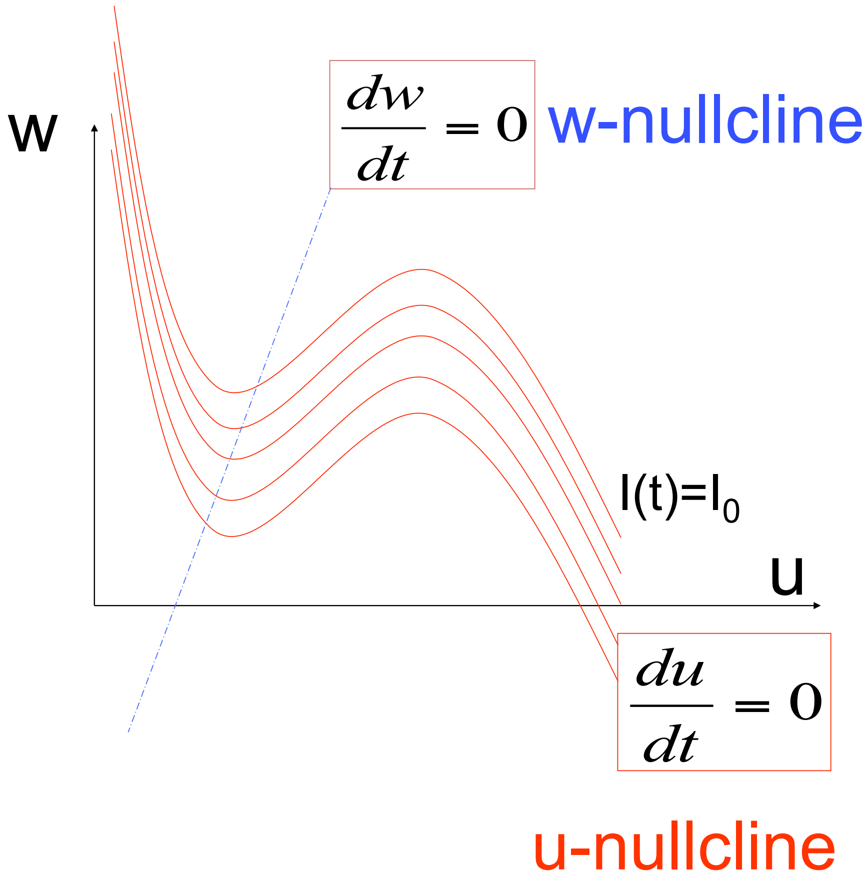
FitzHugh-Nagumo Model: Hopf bifurcation

stimulus

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

apply constant stimulus I_0



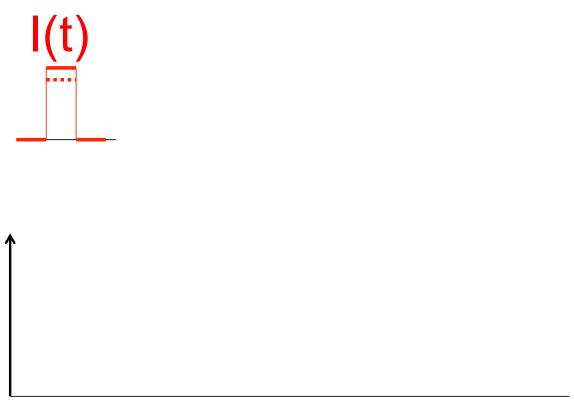
FitzHugh-Nagumo Model - pulse input

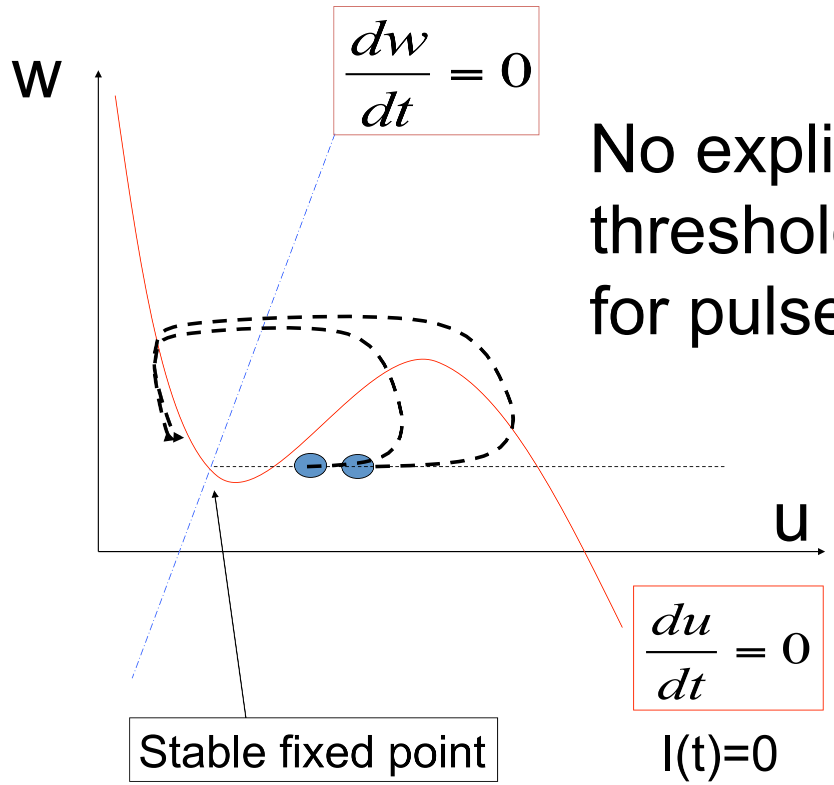
stimulus

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

pulse input





No explicit threshold for pulse input

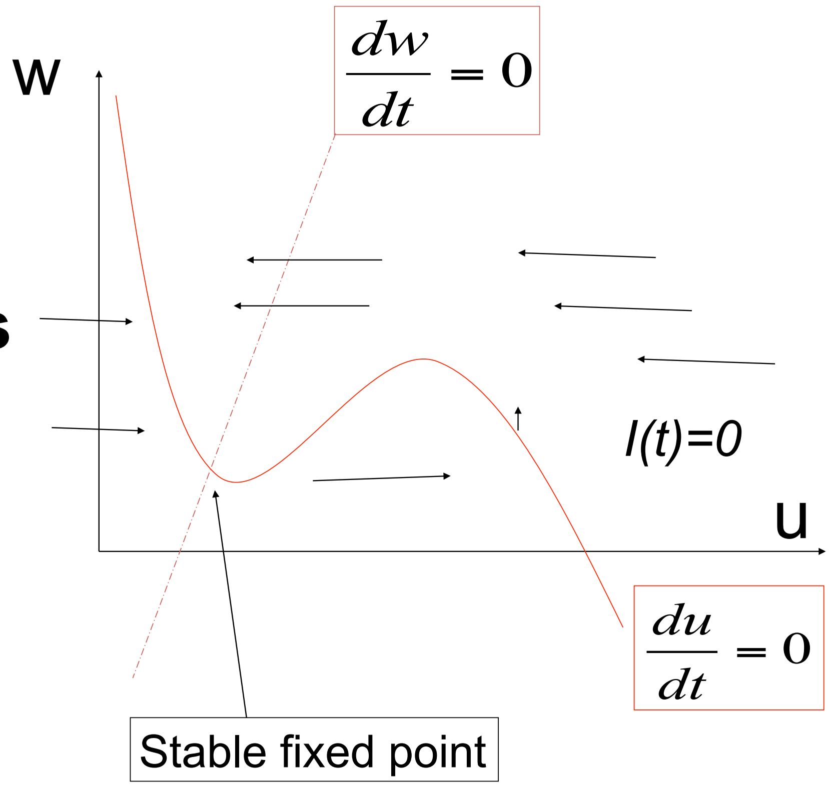
FitzHugh-Nagumo Model - pulse input threshold?

stimulus

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

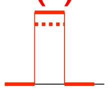
$$\tau_w \frac{dw}{dt} = G(u,w)$$

Separation of time scales

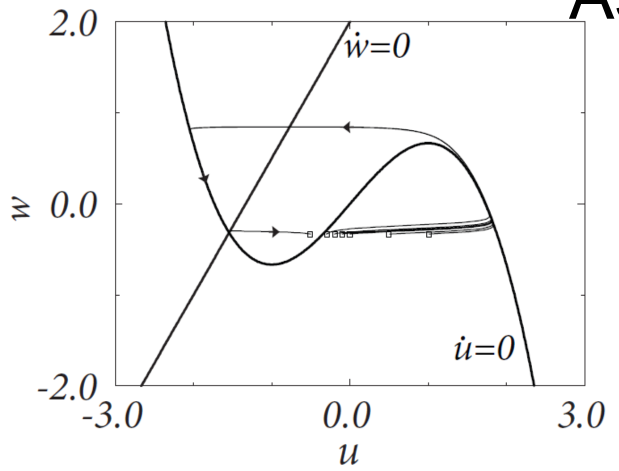


pulse input $\tau_w >> \tau_u$

I(t)

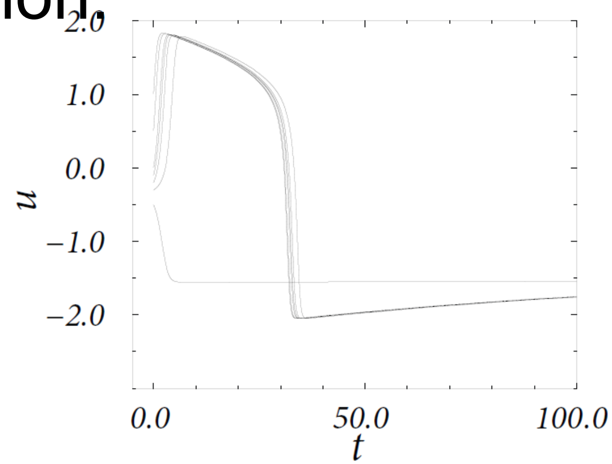


4.4b FitzHugh-Nagumo model: Threshold for Pulse input



Assumption:

$\tau_w >> \tau_u$



Middle branch of u-nullcline
plays role of
'Threshold' (for pulse input)

Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

4.4b Detour: Separation fo time scales in 2dim models

stimulus

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

Assumption:

$$\tau_w >> \tau_u$$

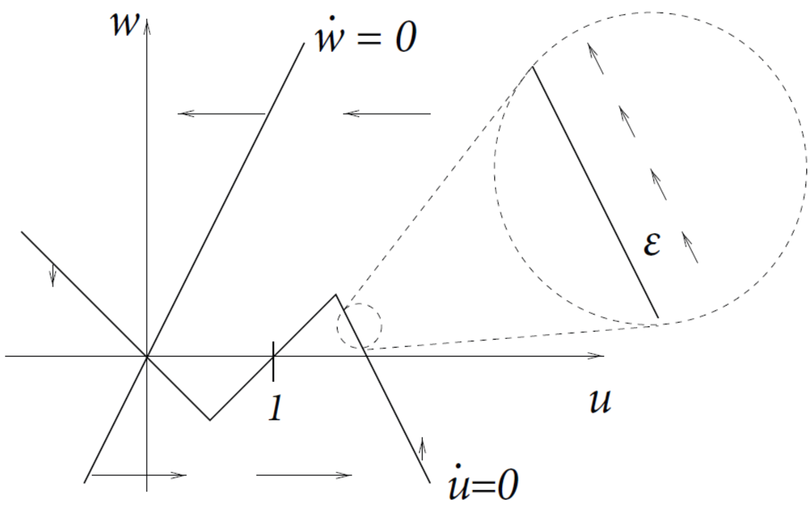
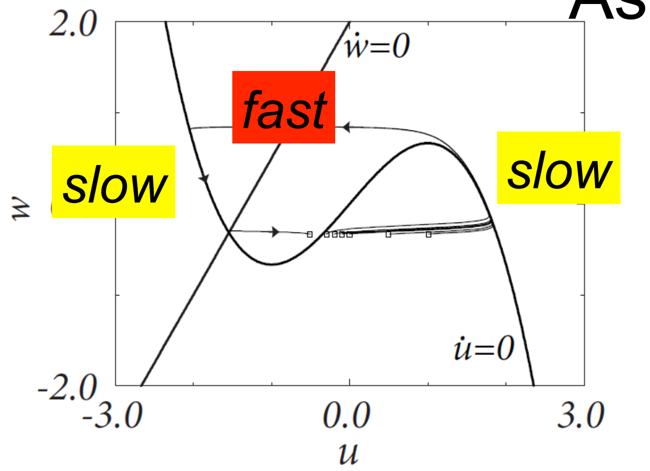


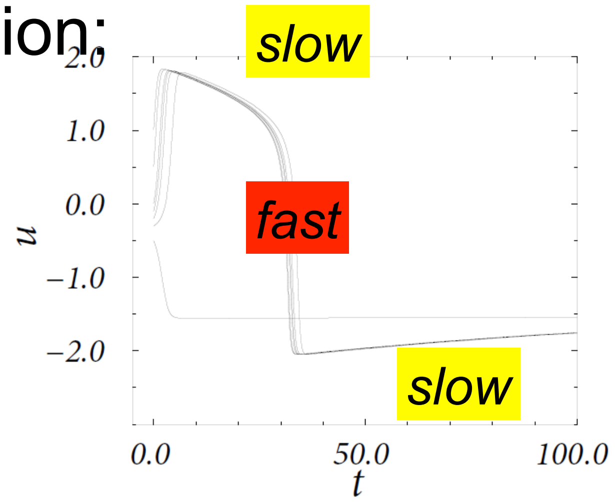
Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

4.4b FitzHugh-Nagumo model: Threshold for Pulse input



Assumption:

$\tau_w >> \tau_u$



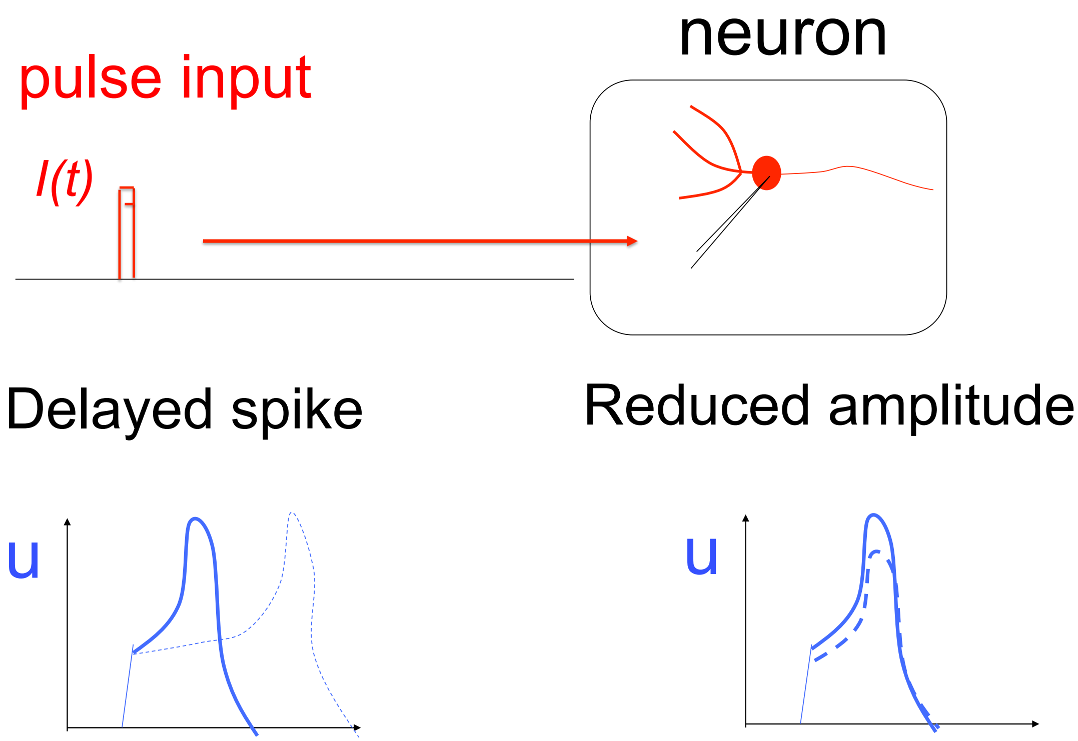
trajectory

-follows u-nullcline: slow

-jumps between branches: fast

Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

Neuronal Dynamics – 4.4b Threshold in 2dim. Neuron Models



Neuronal Dynamics – 4.4 Literature

Reading: W. Gerstner, W.M. Kistler, R. Naud and L. Paninski,
Neuronal Dynamics: from single neurons to networks and models of cognition. Chapter 4*: Introduction*. Cambridge Univ. Press, 2014
OR W. Gerstner and W.M. Kistler, *Spiking Neuron Models*, Ch.3. Cambridge 2002
OR J. Rinzel and G.B. Ermentrout, (1989). Analysis of neuronal excitability and oscillations. In Koch, C. Segev, I., editors, *Methods in neuronal modeling*. MIT Press, Cambridge, MA.

Selected references.

-Ermentrout, G. B. (1996). *Type I membranes, phase resetting curves, and synchrony*. Neural Computation, 8(5):979-1001.
-Fourcaud-Trocme, N., Hansel, D., van Vreeswijk, C., and Brunel, N. (2003). *How spike generation mechanisms determine the neuronal response to fluctuating input.* J. Neuroscience, 23:11628-11640.
-Badel, L., Lefort, S., Berger, T., Petersen, C., Gerstner, W., and Richardson, M. (2008). Biological Cybernetics, 99(4-5):361-370.
- E.M. Izhikevich, Dynamical Systems in Neuroscience, MIT Press (2007)

Neuronal Dynamics – Quiz 4.6.

A. Threshold in a 2-dimensional neuron model with saddle-node bifurcation

[ ] The voltage threshold for repetitive firing is always the same as the voltage threshold for pulse input.

[ ] in the regime below the saddle-node bifurcation, the voltage threshold for repetitive firing is given by the stable manifold of the saddle.

[ ] in the regime below the saddle-node bifurcation, the voltage threshold for repetitive firing is given by the middle branch of the u-nullcline.

[ ] in the regime below the saddle-node bifurcation, the voltage threshold for action potential firing in response to a short pulse input is given by the middle branch of the u-nullcline.

[ ] in the regime below the saddle-node bifurcation, the voltage threshold for action potential firing in response to a short pulse input is given by the stable manifold of the saddle point.

B. Threshold in a 2-dimensional neuron model with subcritical Hopf bifurcation

[ ]in the regime below the bifurcation, the voltage threshold for action potential firing in response to a short pulse input is given by the stable manifold of the saddle point.

[ ] in the regime below the bifurcation, a voltage threshold for action potential firing in response to a short pulse input exists only if $\tau_w >> \tau_u$