Quantum Mechanics & Quantum Computation

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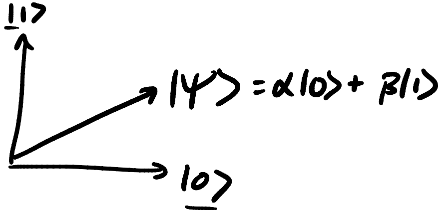
Lecture 15: Spin

Spin as a qubit

Physical Qubits:

1. Initialize
2. Manipulate – gates
3. Measure.



eigenstates.

U Hamiltonian H. , t.

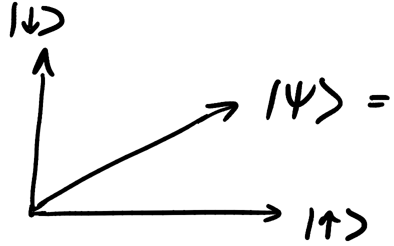
$$U = e^{iHt/\hbar}$$

atomic, photons, spins, quantum dots, superconducting loops.

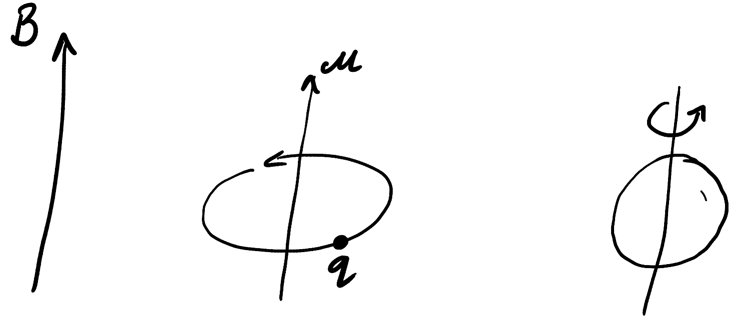
spin

intrinsic angular momentum $\longleftrightarrow$ intrinsic magnetic moment.

$|\uparrow\rangle \qquad |\downarrow\rangle$



$$|\psi\rangle = \underline{\underline{\alpha|\uparrow\rangle + \beta|\downarrow\rangle}}.$$



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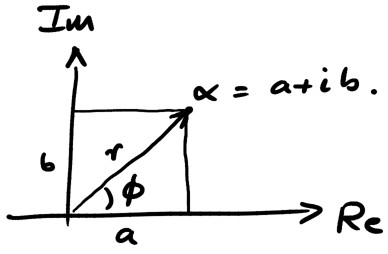
Bloch Sphere

$$|\psi\rangle = \underbrace{\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle}$$

$0 \le \theta \le \pi$

$0 \le \phi \le 2\pi$



$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$\alpha, \beta \in \mathbb{C}$

$|\alpha|^2 + |\beta|^2 = 1.$

$$= r_0 e^{i\phi_0}|0\rangle + r_1 e^{i\phi_1}|1\rangle.$$

$$= e^{i\phi_0}\left[r_0|0\rangle + r_1 e^{i\overbrace{(\phi_1-\phi_0)}^{\phi}}|1\rangle \right]$$

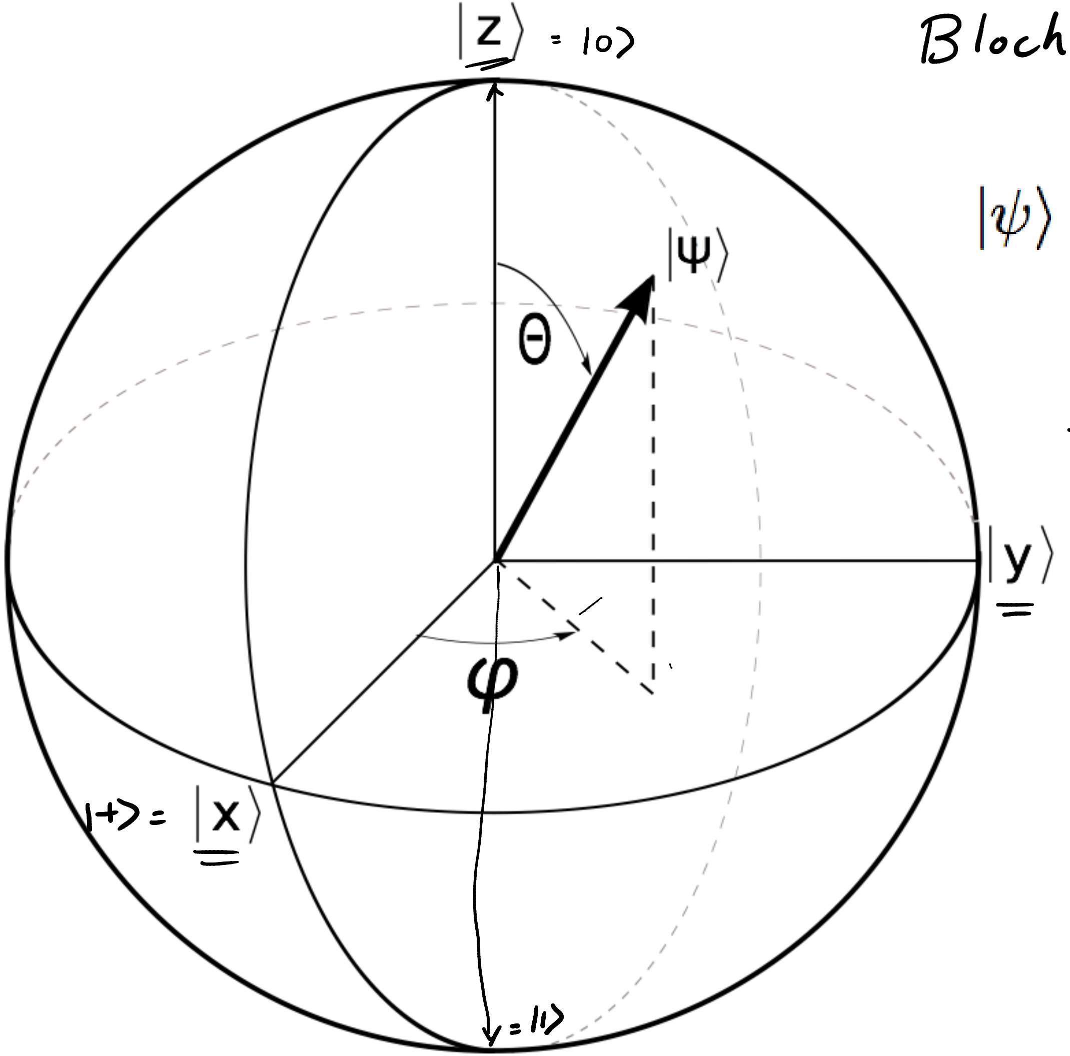
$\alpha = a+ib$

$= re^{i\phi}$

$r_0^2 + r_1^2 = 1.$

$r_0 = \cos\frac{\theta}{2}$

$r_1 = \sin\frac{\theta}{2}.$



Bloch Sphere

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

x : $\theta = \pi/2$ $\phi = 0$.

$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = |+\rangle$.

y : $\theta = \pi/2$ $\phi = \pi/2$.

$\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$

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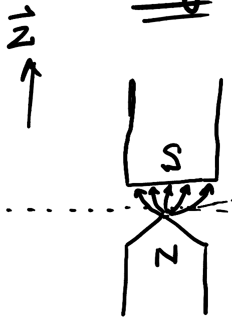
Stern-Gerlach

1922 Stern - Gerlach

Ag



$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$|\uparrow\rangle = |0\rangle$ wp $|\alpha|^2$

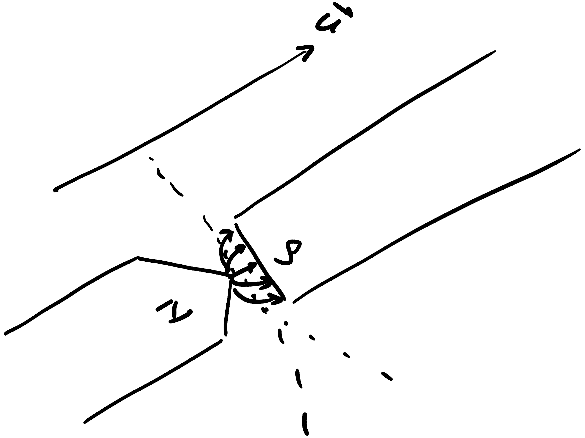
$|\downarrow\rangle = |1\rangle$ wp $|\beta|^2$

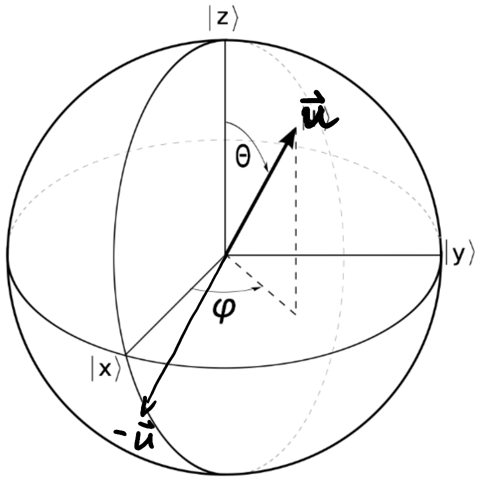
Semi-classical

B $\boxed{-\vec{\mu}\cdot\vec{B}}$

$|\downarrow\rangle$ -ve

$|\uparrow\rangle$ +ve.

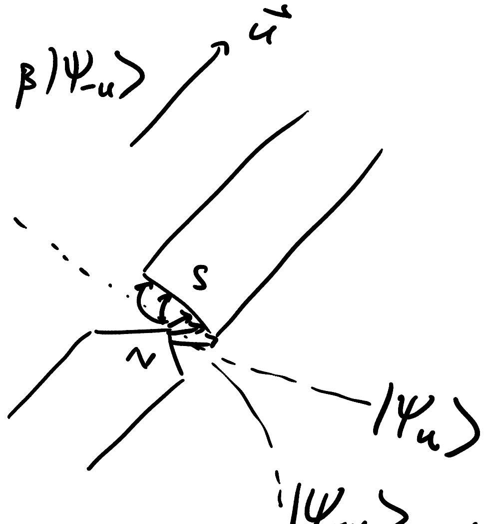




$$|\psi_u\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

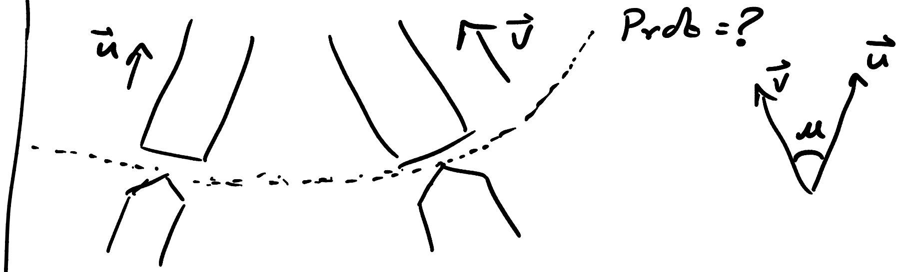
$$|\psi_{-u}\rangle = \sin\frac{\theta}{2}|0\rangle - e^{i\phi}\cos\frac{\theta}{2}|1\rangle$$

$$|\psi\rangle = \alpha|\psi_u\rangle + \beta|\psi_{-u}\rangle$$



wp $|\alpha|^2$

wp $|\beta|^2$.

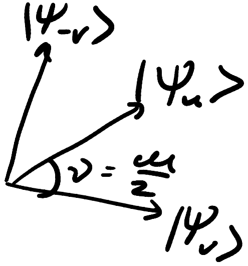


$$\text{Prob} = \frac{1+(\vec{u}\cdot\vec{v})}{2} = \frac{1+\cos\mu}{2}$$

$$= \underline{\underline{\cos^2\frac{\mu}{2}}}.$$

$$\underline{\underline{|\psi_u\rangle}} \longrightarrow \underline{\underline{|\psi_v\rangle}}$$

$$\cos^2\vartheta = \cos^2\frac{\mu}{2}.$$



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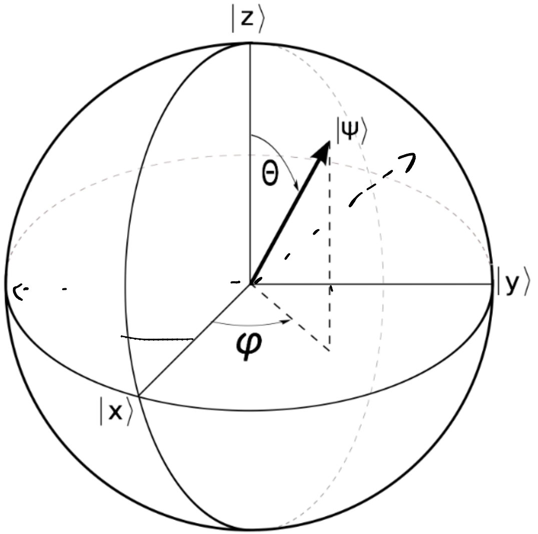
Pauli Spin Matrices



$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

x

$|\psi_x\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \qquad +1$

$|\psi_{-x}\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \qquad -1$

$\sigma_x = X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

z

$|\uparrow\rangle = |0\rangle \longleftrightarrow 1$

$|\downarrow\rangle = |1\rangle \longleftrightarrow -1$

$\longrightarrow y$

$|\psi_y\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle \qquad 1$

$|\psi_{-y}\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle \qquad -1$

$\sigma_z = Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$\sigma_y = Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{bmatrix}$

Pauli spin matrices.