Final Solutions
Principles of Economics for Scientists
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Question 1: Solution

- Solving the utility maximization problem for a typical consumer, we get that his demand function is given by $q^D(p) = A/p$. Summing horizontally over the 1000 consumers we get that aggregate demand in the market is given by $q^D_{mkt}(p) = 1000A/p$.

- Solving the profit maximization problem of a typical firm we get that its supply function is given by $q^S(p) = p$. Summing horizontally over the 10 firms we get that aggregate supply in the market is given by $q^S_{mkt}(p) = 10p$.

- In equilibrium, $q^D_{mkt}(p) = q^S_{mkt}(p)$. Solving the resulting equation we get an equilibrium with $p^* = 10A^{1/2}$ and $q^*_{mkt} = 100A^{1/2}$.

- It follows that the equilibrium profits for every firm are given by $\Pi^* = p^* \frac{q^*_{mkt}}{10} - c\left(\frac{q^*}{10}\right) = 50A$.

Question 2: Solution

- Let τ denote the size of the per-unit sales tax imposed on both consumers and producers.

- Given the results from Question 1, it is straightforward to see that the demand function for each consumer is now given by $q^D(p) = \frac{3}{p+\tau}$. This follows from the fact that the total cost for a consumer of buying one unit of the good is $p + \tau$, where p denotes the market price.

- Summing horizontally, we get that aggregate demand for the market is given by $q^D_{mkt}(p) = \frac{3000}{p+\tau}$.

- Given the results from Question 1, we also see that the supply function for each firm is now given by $q^S(p) = p - \tau$. This follows from the fact that the net price received by the firm for each unit sold is $p - \tau$.

- Summing horizontally, we get that the market's aggregate supply is given by $q^S_{mkt}(p) = 10(p - \tau)$.

- In equilibrium $q^D_{mkt}(p) = q^S_{mkt}(p)$. Solving the resulting equation, and using the fact that $\tau = 10$ \$/unit, we get an equilibrium with $p^* = 20$ and $q^*_{mkt,\tau=10} = 100$.

- Computing the equilibrium in the absence of any taxes, which can be done using the previous equations but setting $\tau = 0$, we get an equilibrium with $q^*_{mkt,\tau=0} = 100\sqrt{3}$.

- This equilibrium is efficient by the First Welfare Theorem.

- The aggregate marginal benefit curve for the market is given by the inverse of aggregate demand with no taxation, which is given by $3000/q$.

- Likewise, the aggregate marginal cost curve is given by $q/10$.

- It follows that the DWL is given by

$$\int_{100}^{100\sqrt{3}} \left(\frac{3000}{q} - \frac{q}{10}\right) dq = 3000 log(3^{1/2}) - 1000$$

Question 3: Solution

- From the solutions to Question 1, and using the fact that $A = 3$, we know that the utility of a consumer without the tax is given by $3\log\left(\frac{\sqrt{3}}{10}\right) + W - 3$, where W denotes the endowment of \$ of each consumer

- From the solutions to Question 2, we get that the total revenue raised by the taxes is $100(10 + 10) = 2000$. Thus, each consumer gets back a lump-sum transfer of \$2 under the policy.

- From the solutions to Question 2, we also get that the utility of a consumer with the tax (and after the lump-sum transfer) is given by $3\log\left(\frac{1}{10}\right) + W - 3 + 2$

- Thus, the introduction of the policy changes the consumer's utility by $2 - 3\log(\sqrt{3})$, which is a positive numer.
- Since the consumers are made better off by the policy, we expect them to favor it

Question 4: Solution

- Let q denote the total level of consumption in the market.
- Then the marginal total externality is given by $\frac{d}{dq}\left(\frac{q^2}{20000}\right)1000 = \frac{q}{10}$
- Given this, the marginal social benefit of q is given by $\frac{3000}{q} - \frac{q}{10}$
- The marginal social cost is given by $\frac{q}{10}$
- From this, it follows that the optimal level of q is given by equating the marginal social benefit and marginal social cost, and solving for q. The solution is $q^{opt} = 100\sqrt{\frac{3}{2}}$
- We know that the optimal Pigouvian tax equals the marginal damage at the optimum, which is given by

$$\frac{q^{opt}}{10} = 10\sqrt{\frac{3}{2}}$$

Question 5: Solution

- The market equilibrium quantity in the absence of permits is $q^* = 100\sqrt{3}$, which means that the introduction of the permits has no effect on the equilibrium level of production.
- From the answer to Question 4, we know that $q^{opt} = 100\sqrt{\frac{3}{2}}$
- It follows that the DWL is given by the integral,

$$\int_{q^{opt}}^{q^*} MSB(q) - MSC(q)dq = \int_{100\sqrt{\frac{3}{2}}}^{100\sqrt{3}} (\frac{3000}{q} - \frac{q}{10} - \frac{q}{10})dq$$

which equals $3000\left(\frac{1}{2} + \log\left(\sqrt{\frac{1}{2}}\right)\right)$.

Question 6: Solution

- Compute the optimal level of total production in the market for good q. To do this, use the fact that marginal social benefit is given by $1000-2q$, that marginal social cost is given by 100, and equate the two to get $q^{opt} = 450$

- Given the CRS cost function, we know that the equilibrium price in a competitive market is $p^* = MC = 100$. Under the given market demand function, this implies that $q^* = 900$.

- Next, let's compute the equilibrium in the monopoly case, which is given by the solution to the monopolist's profit maximiation problem given by $max_{q\geq 0} qp^D(q) - c(q) = q(1000-q) - 100q$. The solution is $q^{mon} = 450$.

- It follows that the DWL is positive in the competitive case, but zero under monopoly. Thus, the DWL is larger in the case of perfect competition.