Week 4 – part 3: Analysis of a 2D neuron model

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Neuronal Dynamics:
Computational Neuroscience of Single Neurons

Week 4 – Reducing detail:
Two-dimensional neuron models

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Week 4 – part 3: Analysis of a 2D neuron model

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Neuronal Dynamics – 4.3. Analysis of a 2D neuron model

2-dimensional equation

stimulus

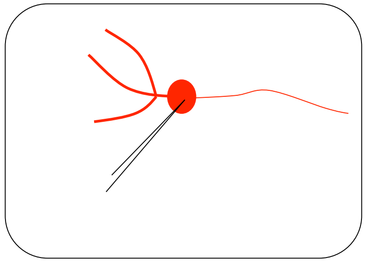
$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

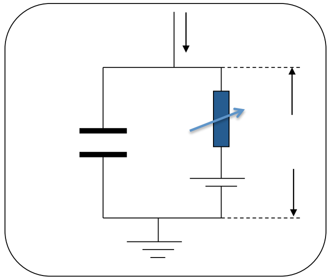
$$\tau_w \frac{dw}{dt} = G(u,w)$$

Enables graphical analysis!

- Pulse input
- Constant input

Neuronal Dynamics – 4.3. 2D neuron model : Pulse input





$$\tau \cdot \frac{d}{dt} u = F(u,w) + RI$$

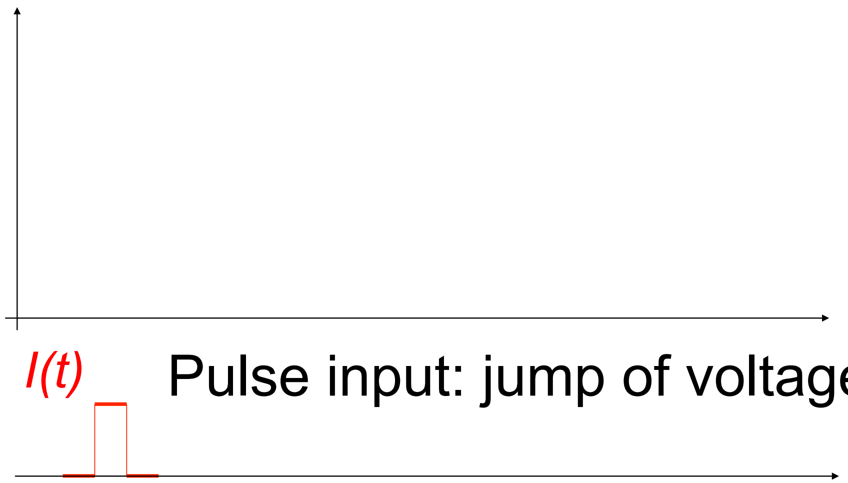
$$\tau_w \frac{d}{dt} w = G(u,w)$$

pulse input

Neuronal Dynamics – 4.3. FitzHugh-Nagumo Model : Pulse input

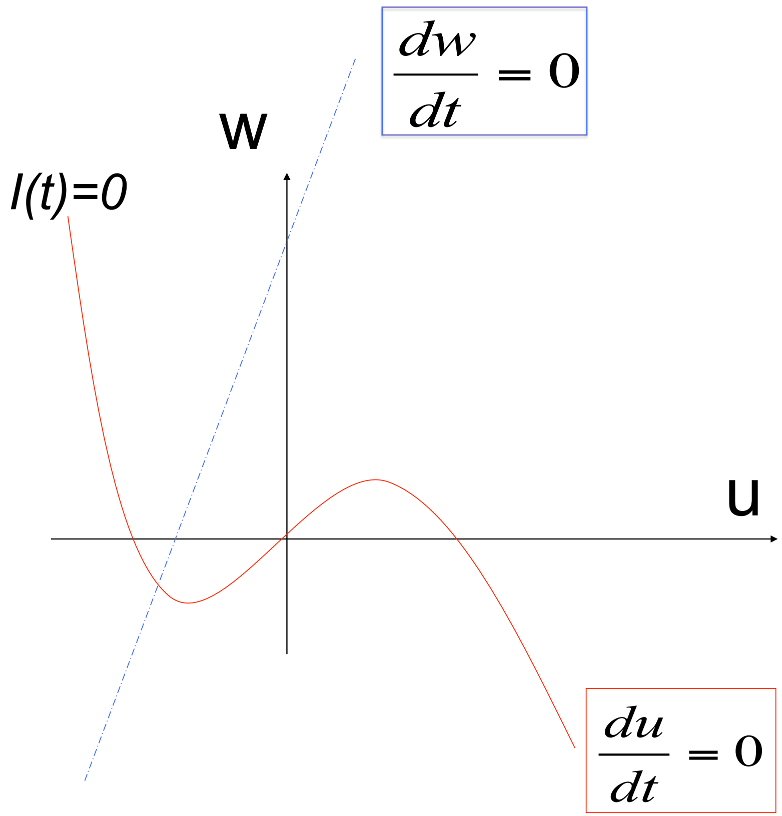
$$\tau \frac{du}{dt} = F(u,w) + RI(t) = u - \frac{1}{3}u^3 - w + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w) \qquad = b_0 + b_1 u - w$$

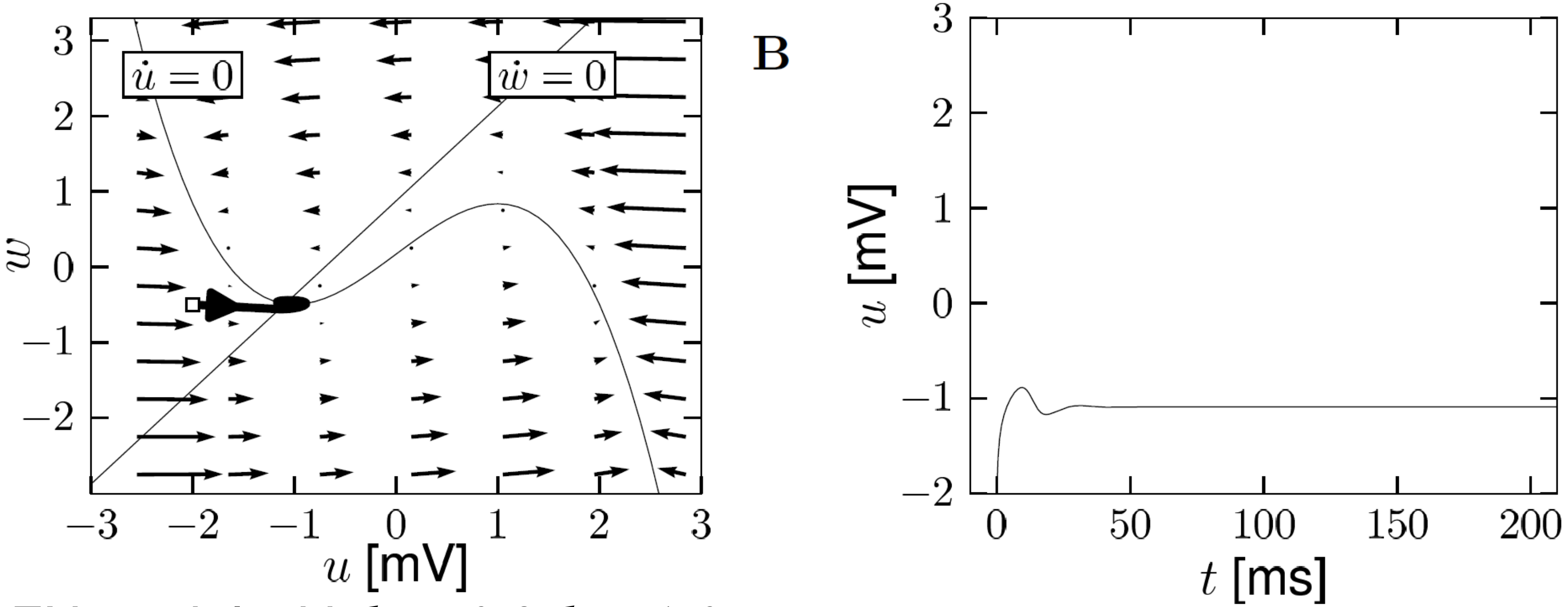


pulse input

Pulse input: jump of voltage



Neuronal Dynamics – 4.3. FitzHugh-Nagumo Model : Pulse input



FN model with $b_0 = 0.9; b_1 = 1.0$

Pulse input: jump of voltage/initial condition

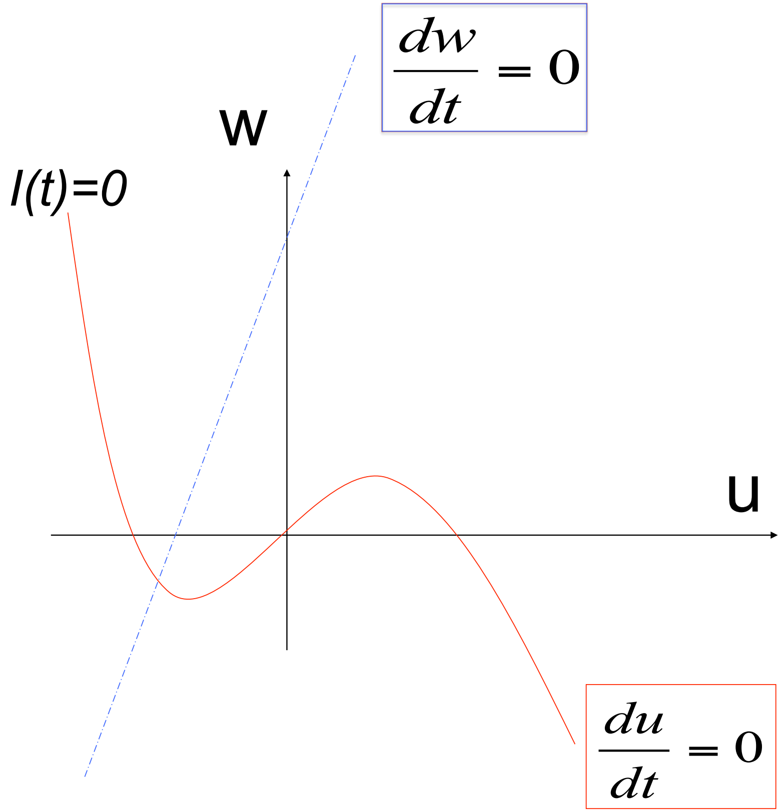
Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014

Neuronal Dynamics – 4.3. FitzHugh-Nagumo Model

Pulse input:
- jump of voltage
- 'new initial condition'
- spike generation for large input pulses

constant input:
- graphics?
- spikes?
- repetitive firing?

Now!



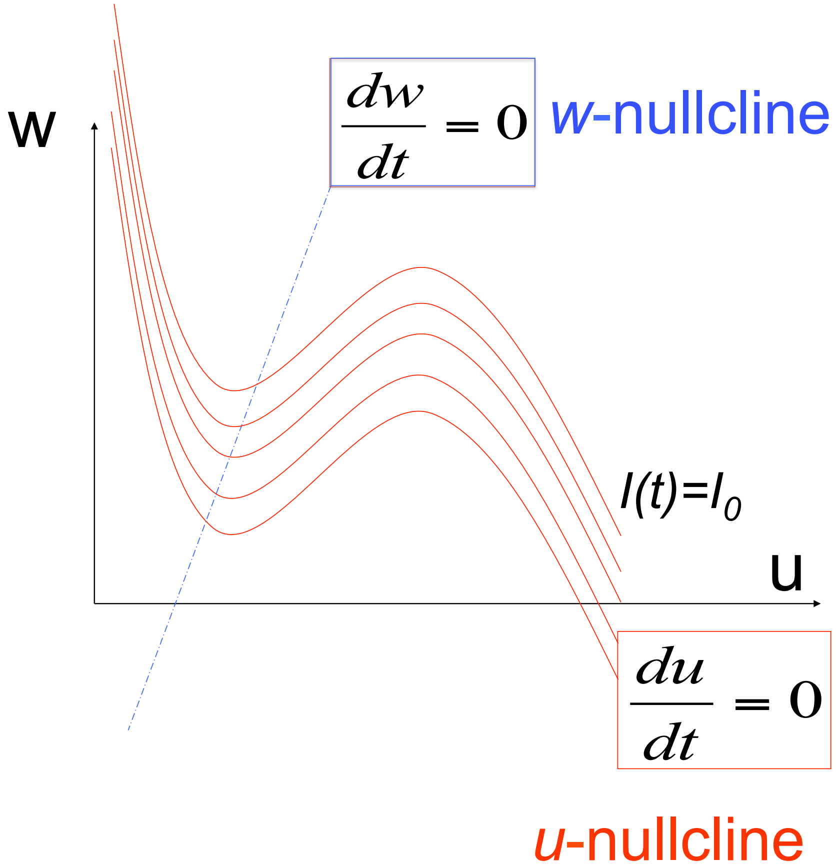
Neuronal Dynamics – 4.3. FitzHugh-Nagumo Model: Constant input

$$\tau \frac{du}{dt} = F(u,w) + RI_0$$

$$= u - \frac{1}{3}u^3 - w + RI_0$$

$$\tau_w \frac{dw}{dt} = G(u,w) = b_0 + b_1 u - w$$

Intersection point (fixed point)
-moves
-changes Stability



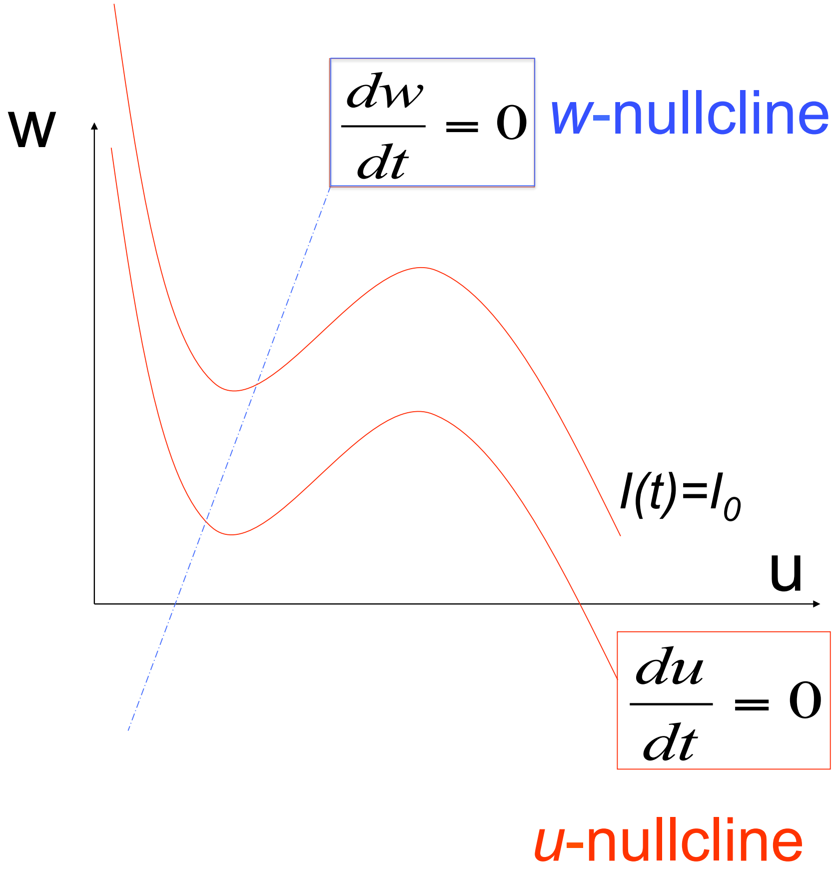
Neuronal Dynamics – 4.3. FitzHugh-Nagumo Model: Constant input

$$\tau \frac{du}{dt} = F(u,w) + RI_0$$

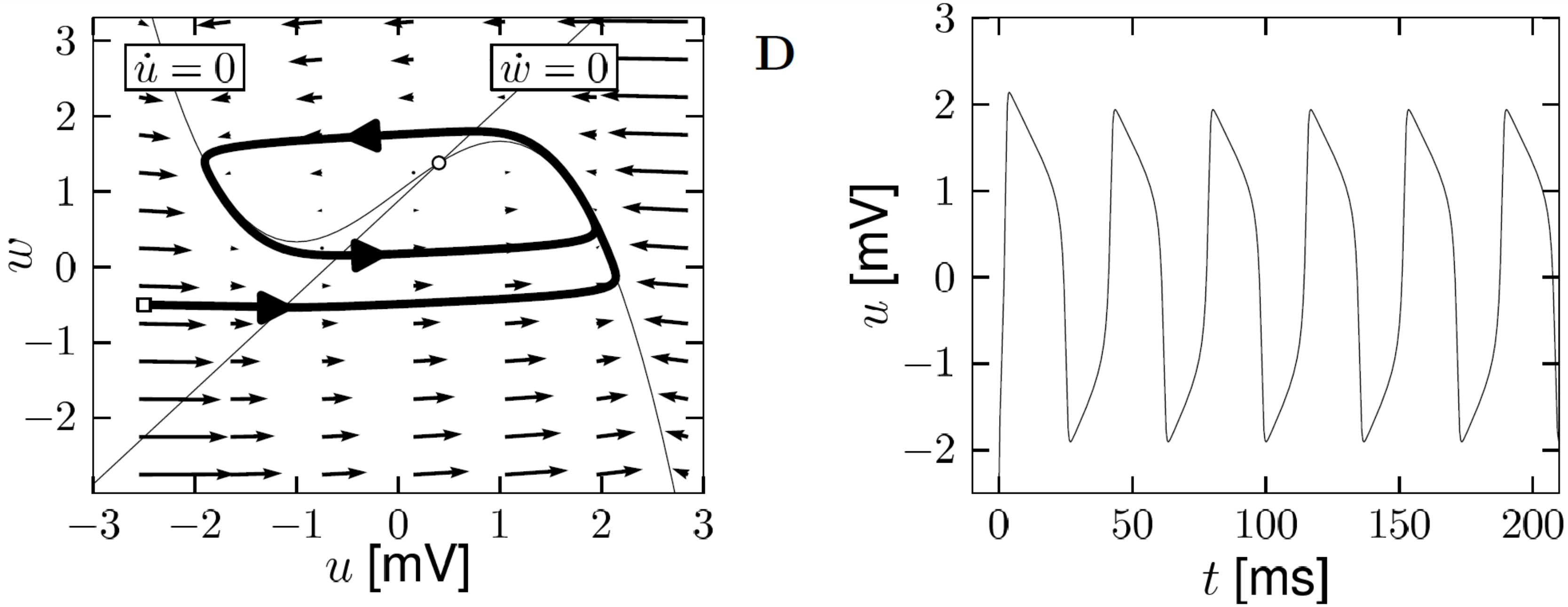
$$= u - \frac{1}{3}u^3 - w + RI_0$$

$$\tau_w \frac{dw}{dt} = G(u,w) = b_0 + b_1 u - w$$

Intersection point (fixed point)
-moves
-changes Stability



Neuronal Dynamics – 4.3. FitzHugh-Nagumo Model : Constant input



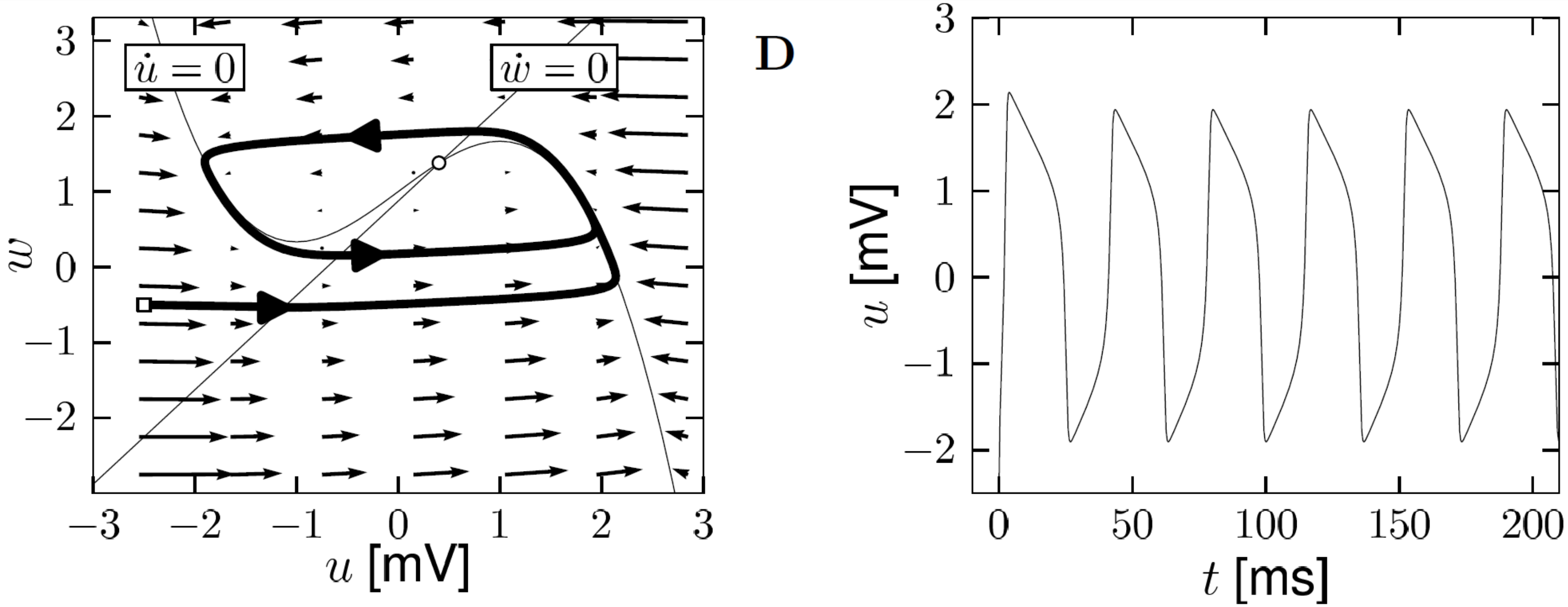
FN model with $b_0 = 0.9; b_1 = 1.0; RI_0 = 2$

constant input: u-nullcline moves

limit cycle

Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

Neuronal Dynamics – 4.3. Limit Cycle



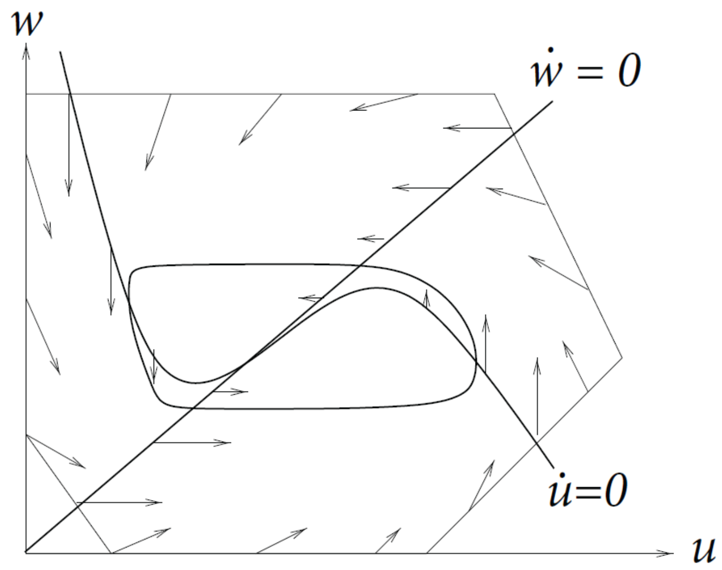
-unstable fixed point in 2D

-bounding box with inward flow

→ limit cycle *(Poincare Bendixson)*

Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

Neuronal Dynamics – 4.3. Limit Cycle



In 2-dimensional equations,
a limit cycle must exist, if we can
find a surface

-containing one unstable fixed point
-bounding box with inward flow
→ limit cycle *(Poincare Bendixson)*

Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

Neuronal Dynamics – 4.3. Analysis of a 2D neuron model

2-dimensional equation

stimulus ↓

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

Enables graphical analysis!

-Pulse input

→ AP firing (or not)

- Constant input

→ repetitive firing (or not)

Neuronal Dynamics – Quiz 4.5.

A. Short current pulses. In a 2-dimensional neuron model, the effect of a delta current pulse can be analyzed

[ ] By moving the u-nullcline vertically upward
[ ] By moving the w-nullcline vertically upward
[ ] As a potential change in the stability or number of the fixed point(s)
[ ] As a new initial condition
[ ] By following the flow of arrows in the appropriate phase plane diagram

B. Constant current. In a 2-dimensional neuron model, the effect of a constant current can be analyzed

[ ] By moving the u-nullcline vertically upward
[ ] By moving the w-nullcline vertically upward
[ ] As a potential change in the stability or number of the fixed point(s)
[ ] As a new initial condition
[ ] By following the flow of arrows in the appropriate phase plane diagram