Week 4 – part 1 : Separation of time scales

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Neuronal Dynamics:
Computational Neuroscience of Single Neurons

Week 4 – Reducing detail:
Two-dimensional neuron models

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Week 4 – part 1 : Separation of time scales

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4.1 **From Hodgkin-Huxley to 2D**

- Overview: From 4 to 2 dimensions
- MathDetour 1: Separation of time scales
- MathDetour 2: Exploiting similarities

4.2 **Phase Plane Analysis**

- role of nullclines

4.3 **Analysis of a 2D Neuron Model**

- MathDetour 3: Stability of fixed points

4.4 **Type I and II Neuron Models**

- where is the firing threshold?
- separation of time scales

4.5. **Nonlinear Integrate-and-fire**

- from two to one dimension

Neuronal Dynamics – MathDetour 4.1: Separation of time scales

Two coupled differential equations

$$\tau_1 \frac{dx}{dt} = -x + c(t)$$

$$\tau_2 \frac{dy}{dt} = f(y) + g(x)$$

Separation of time scales

$$\tau_1 = \tau_2$$

Reduced 1-dimensional system

$$\tau_2 \frac{dy}{dt} = f(y) + g(c(t))$$

Neuronal Dynamics – MathDetour 4.1: Separation of time scales

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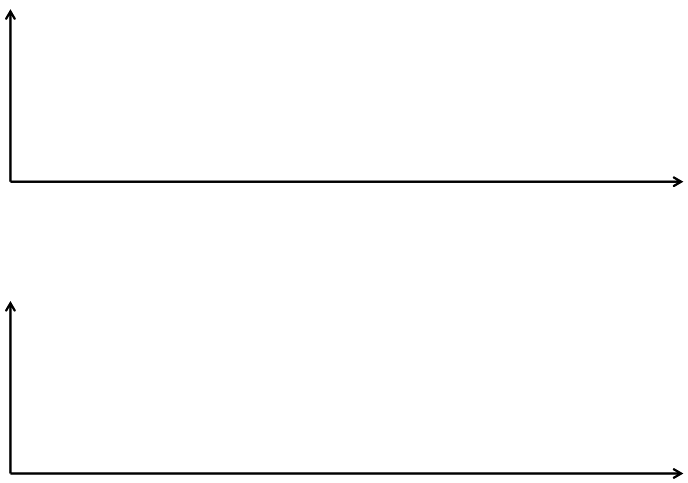
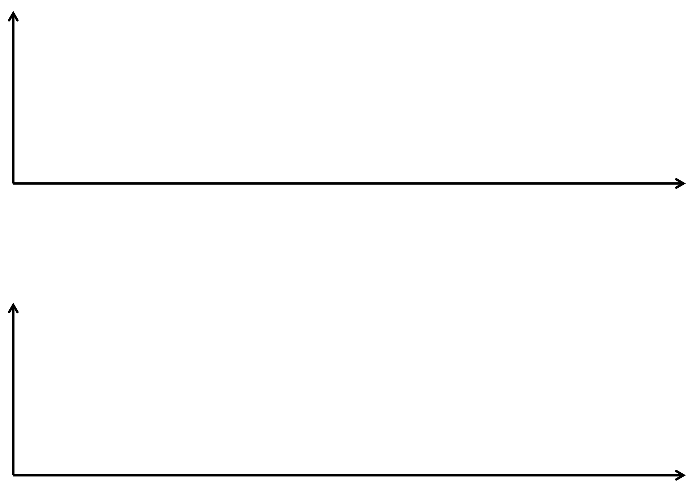
Neuronal Dynamics – MathDetour 4.1: Separation of time scales

Linear differential equation $\tau_1 \frac{dx}{dt} = -x + c(t)$



step



‘slow drive’

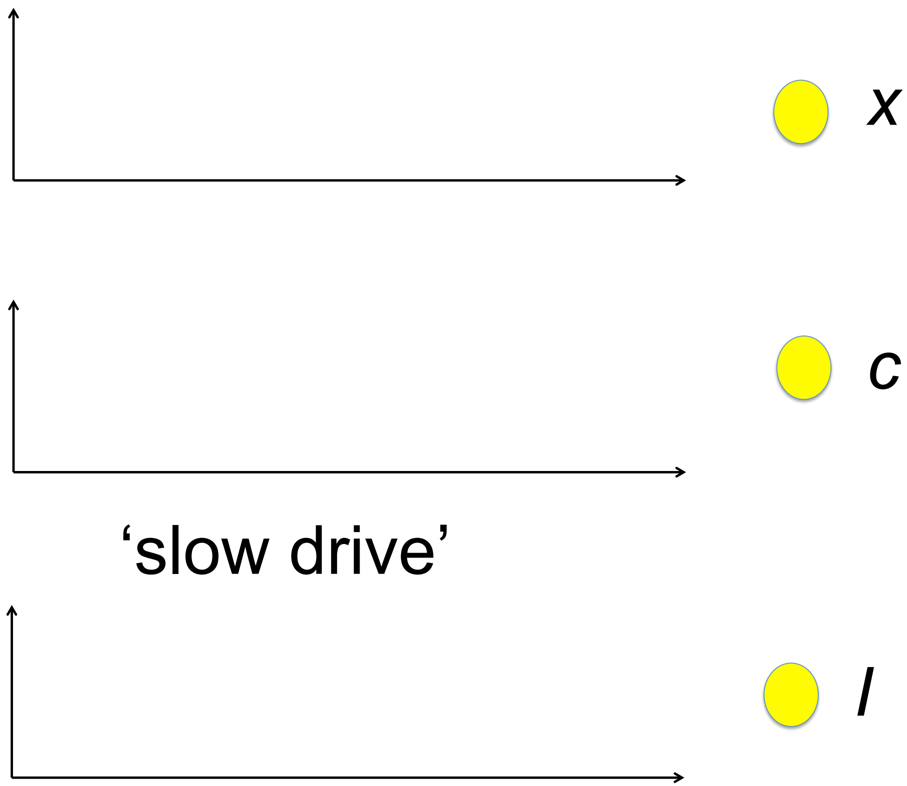
Neuronal Dynamics – MathDetour 4.1: Separation of time scales

Two differential equations

$$\tau_1 \frac{dx}{dt} = -x + c(t)$$

$$\tau_2 \frac{dc}{dt} = -c + I(t)$$

$$\tau_1 = \tau_2$$



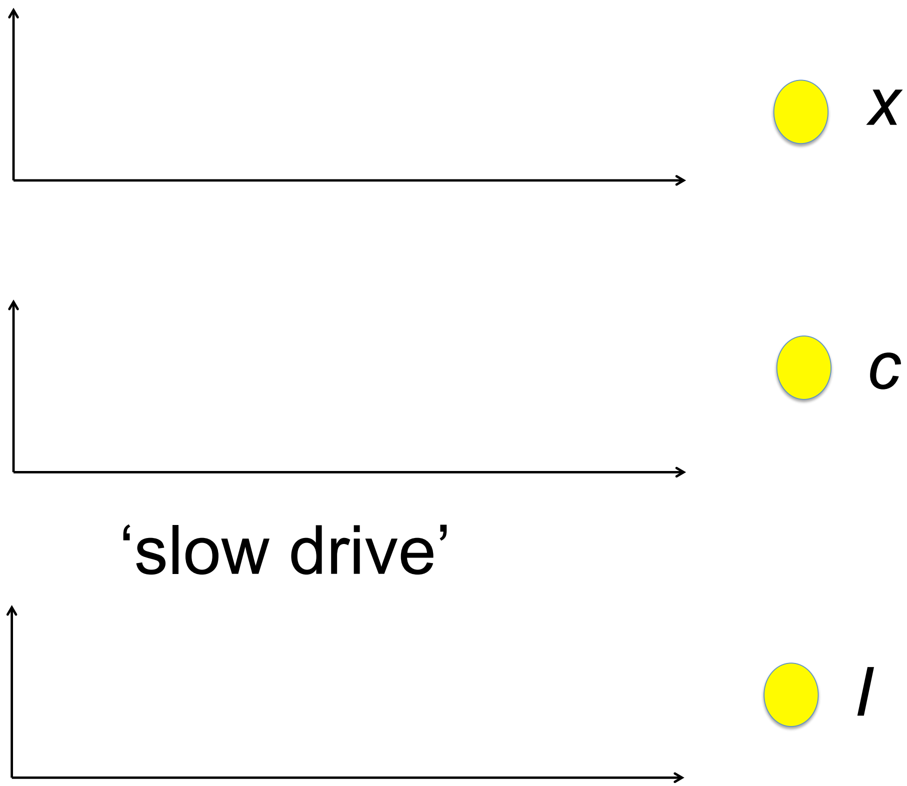
Neuronal Dynamics – MathDetour 4.1: Separation of time scales

Two coupled differential equations

$$\tau_1 \frac{dx}{dt} = -x + c(t)$$

$$\tau_2 \frac{dc}{dt} = -c + f(x) \quad +I(t)$$

$$\tau_1 = \tau_2$$



Neuronal Dynamics – Reduction of Hodgkin-Huxley model

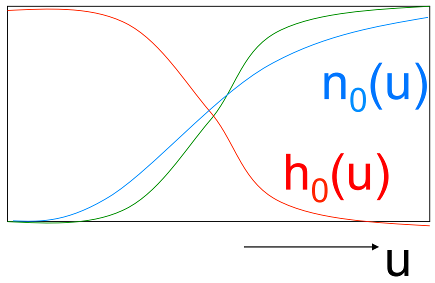
stimulus

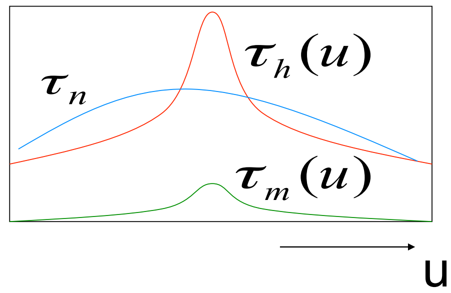
$$C\frac{du}{dt} = \overbrace{-g_{Na}m^3h(u-E_{Na})}^{I_{Na}} \overbrace{-g_K n^4(u-E_K)}^{I_K} \overbrace{-g_l(u-E_l)}^{I_{leak}} + I(t)$$

$$\frac{dm}{dt} = -\frac{m-m_0(u)}{\tau_m(u)}$$

$$\frac{dh}{dt} = -\frac{h-h_0(u)}{\tau_h(u)}$$

$$\frac{dn}{dt} = -\frac{n-n_0(u)}{\tau_n(u)}$$





dynamics of *m* is fast $\longrightarrow$ $m(t) = m_0(u(t))$

Fast compared to what?

Neuronal Dynamics – MathDetour 4.1: Separation of time scales

Two coupled differential equations

$$\tau_1 \frac{dx}{dt} = -x + h(y)$$

$$\tau_2 \frac{dy}{dt} = f(y) + g(x)$$

Separation of time scales

$$\tau_1 = \tau_2 \quad \rightarrow x = h(y)$$

Reduced 1-dimensional system

$$\tau_2 \frac{dy}{dt} = f(y) + g(h(y))$$

Neuronal Dynamics – Quiz 4.2.

A- Separation of time scales:
We start with two equations

$$\tau_1 \frac{dx}{dt} = -x + y + I(t)$$

$$\tau_2 \frac{dy}{dt} = -y + x^2 + A$$

[ ] If $\tau_1 = \tau_2$ then the system can be reduded to

$$\tau_2 \frac{dy}{dt} = -y + [y + I(t)]^2 + A$$

[ ] If $\tau_2 = \tau_1$ then the system can be reduded to

$$\tau_1 \frac{dx}{dt} = -x + x^2 + A + I(t)$$

[ ] None of the above is correct.

B- Separation of time scales:
A channel with gating variable *r*, given by

$$\tau_1 \frac{dr}{dt} = -r + r_0(u)$$

influences the voltage

$$\tau_2 \frac{du}{dt} = -(u - u_0) + r^2 A$$

We assume that $\tau_1 = \tau_2$

In this case a reduction of dimensionality
[ ] is not possible
[ ] is possible and the result is

$$\tau_2 \frac{du}{dt} = -u + u_0 + [r_0(u)]^2 A$$

[ ] is possible and the result is

$$\tau_1 \frac{dr}{dt} = -r + r_0(u_0 + r^2 A)$$