Week 7 – part 5 : Parameter estimation

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Neuronal Dynamics:
Computational Neuroscience of Single Neurons

Week 7 – Optimizing Neuron Models For Coding and Decoding

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Week 7 – part 5 : Parameter estimation

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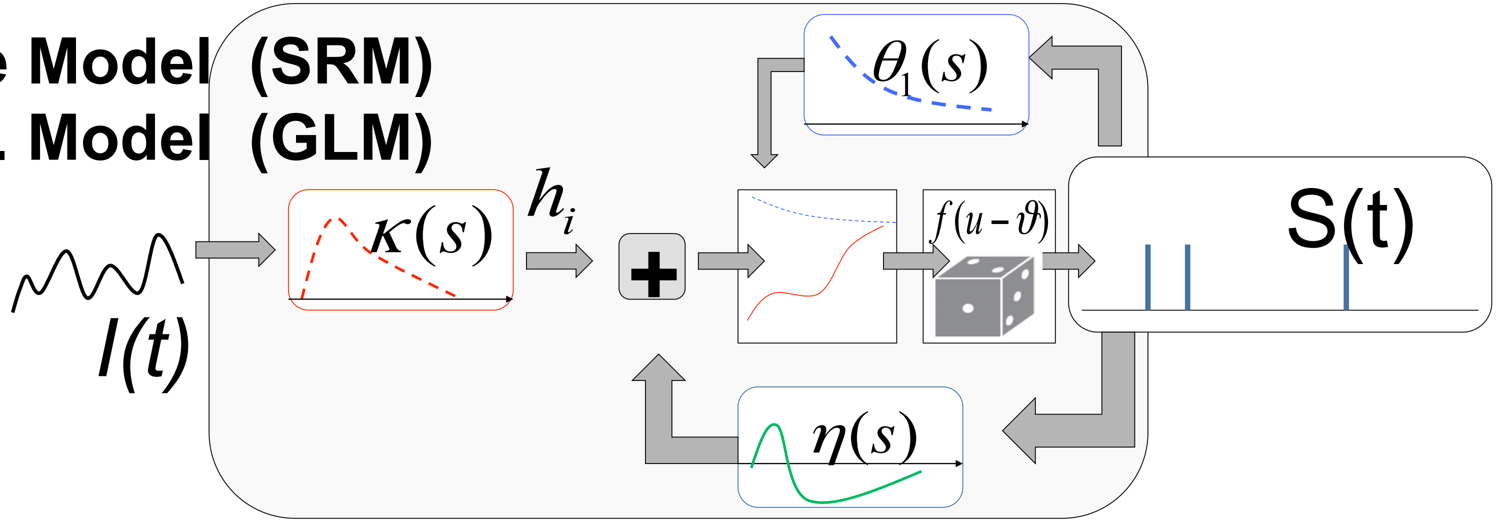
Neuronal Dynamics – 7.5 Parameter estimation: voltage

Spike Response Model (SRM)
Generalized Lin. Model (GLM)



Subthreshold potential

$$u(t) = \int \eta(s) S(t-s) ds + \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$$

known spike train — known input

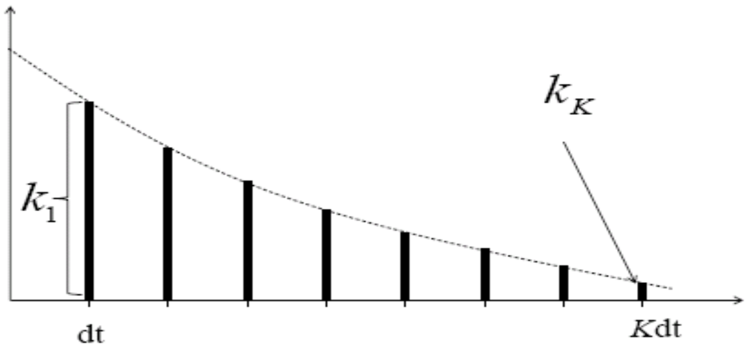
Linear filters/linear in parameters

Neuronal Dynamics – 7.5 Parameter estimation: voltage

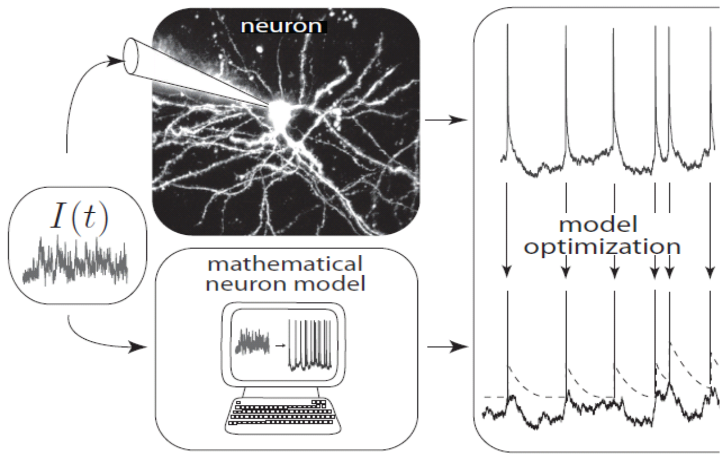
Linear in parameters = linear fit = quadratic problem

$$u(t) = \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$$

$$u(t_n) = \sum k_k I_{n-k} + u_{rest}$$



comparison model-data

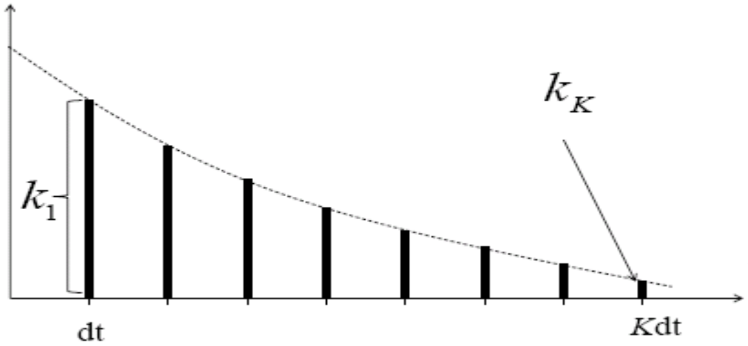


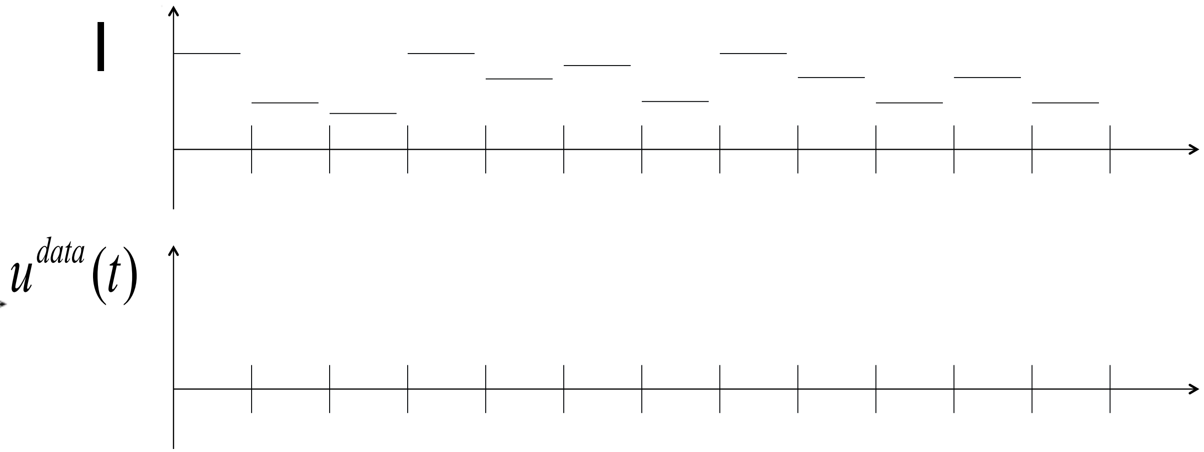
Neuronal Dynamics – 7.5 Parameter estimation: voltage

Linear in parameters = linear fit

$$u(t) = \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$$

$$u(t_n) = \sum k_k I_{n-k} + u_{rest}$$





$$E = \sum_n [u^{data}(t_n) - \sum_{k=1}^{K} k_k I_{n-k} - u_{rest}]^2$$

Neuronal Dynamics – 7.5 Parameter estimation: voltage

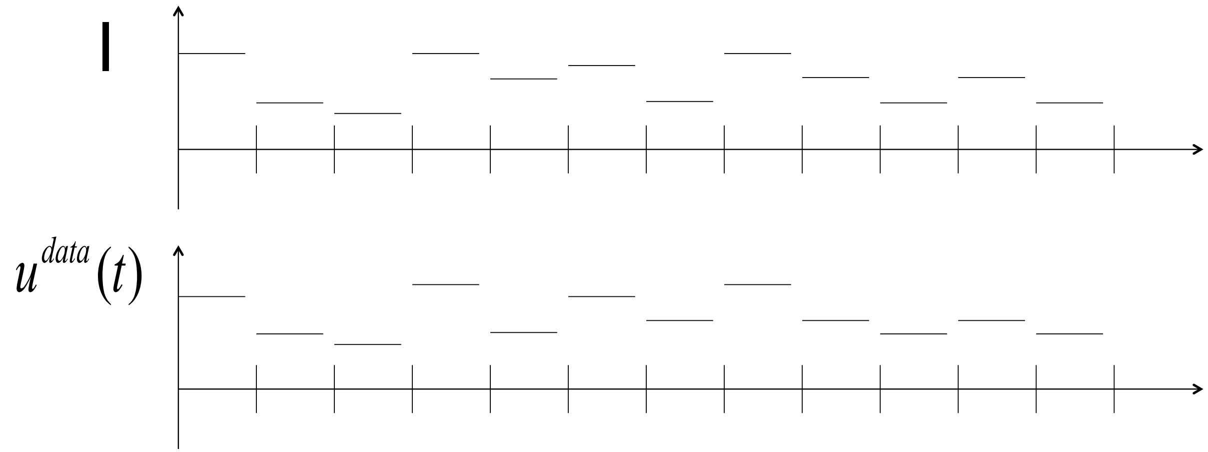
Linear in parameters = linear fit = quadratic optimization

Model

$$u(t) = \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$$

$$u(t_n) = \sum_k k_k \, I_{n-k} + u_{rest}$$

Data $u^{data}(t)$

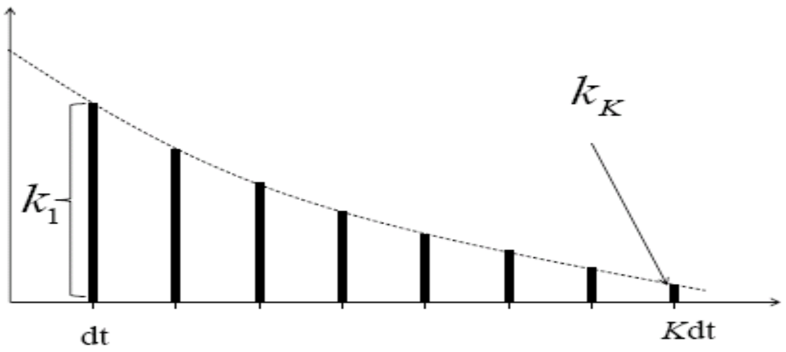


$$E = \sum_n [u^{data}(t_n) - \sum_{k=1}^{K} k_k \, I_{n-k} - u_{rest}]^2$$

Neuronal Dynamics – 7.5 Parameter estimation: voltage

Vector notation

$$u(t_n) = \sum_k k_k I_{n-k} + u_{rest}$$



$$u(t_n) = \vec{k} \cdot \vec{x}_n$$

$$E = \sum_n [u^{data}(t_n) - \sum_{k=1}^{K} k_k I_{n-k} - u_{rest}]^2$$

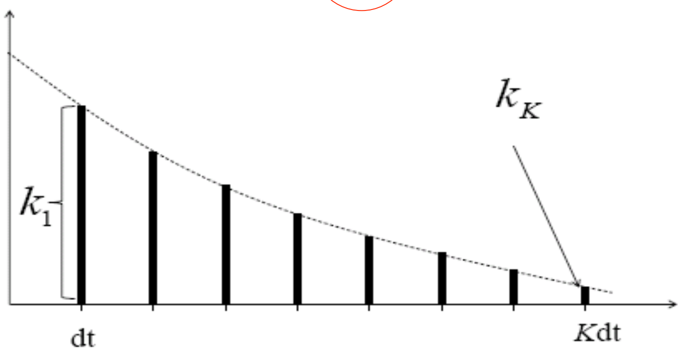
Neuronal Dynamics – 7.5 Parameter estimation: voltage

Linear in parameters = linear fit = quadratic problem

$$u(t) = \int_0^\infty \kappa(s) I(t-s) ds + u_{rest} + \int_0^\infty \eta(s) S(t-s) ds$$

$$u(t_n) = \sum k_k I_{n-k} + u_{rest}$$

$$u(t_n) = \vec{k} \cdot \vec{x}_n$$



$$\begin{array}{c|c} {}_{\text{time}} \diagdown^{\text{input } \vec{\mathbf{x}}} & x_1 \quad x_2 \quad x_3 \quad \cdots \quad x_K \\ \hline \begin{array}{l} t=K+1 \\ t=K+2 \\ t=K+3 \\ \cdot \\ \cdot \\ \cdot \\ t=T \end{array} & \left(\begin{array}{ccccc} I_K & I_{K-1} & I_{K-2} & \cdots & I_1 \\ I_{K+1} & I_K & I_{K-1} & \cdots & I_2 \\ I_{K+2} & I_{K+1} & I_K & \cdots & I_3 \\ & & & & \\ & & & & \\ & & & & \\ I_T & I_{T-1} & I_{T-2} & \cdots & I_{T-K+1} \end{array} \right) dt \end{array}$$

$$E = \sum_n [u^{data}(t_n) - \sum_{k=1}^{K} k_k I_{n-k} - u_{rest}]^2$$

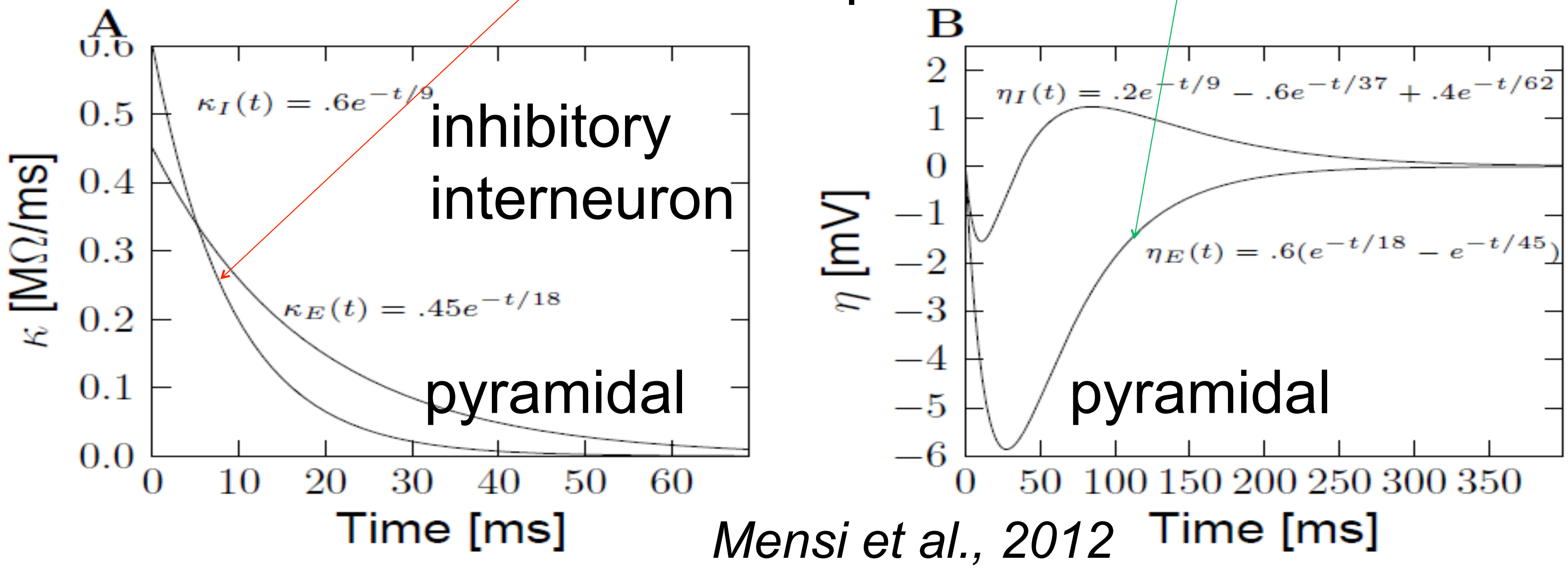
Neuronal Dynamics – 7.5 Extracted parameters: voltage

Subthreshold potential

$$u(t) = \int_0^\infty \kappa(s) I(t-s)\,ds + u_{rest} + \int \eta(s) S(t-s)\,ds$$

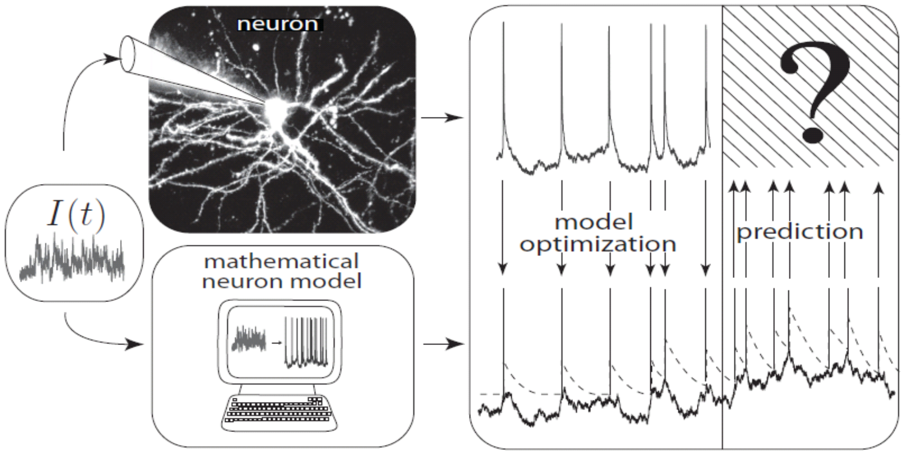
known input

known spike train



Mensi et al., 2012

Neuronal Dynamics – What is a good neuron model?



A) Predict spike times
B) Predict subthreshold voltage
C) Easy to interpret (not a 'black box')
D) Flexible
E) Systematic: 'optimize' parameters

Week 7 – part 5b : Parameter estimation for spike times

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Week 7 – part 5b : Parameter estimation for spike times

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7.1 What is a good neuron model?
- Models and data

7.2 AdEx model
- Firing patterns and analysis

7.3 Spike Response Model (SRM)
- Integral formulation

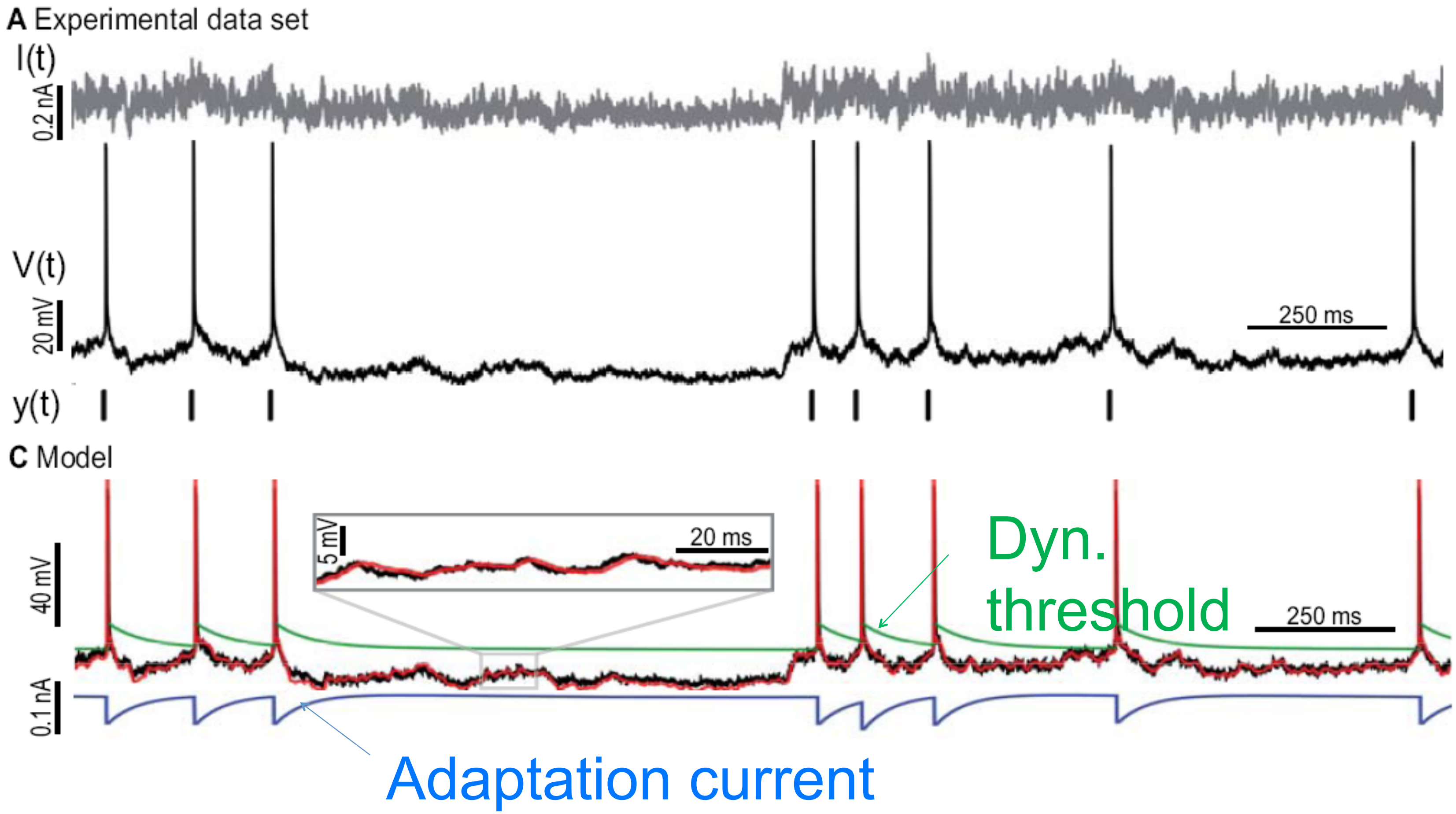
7.4 Generalized Linear Model (GLM)
- Adding noise to the SRM

7.5 Parameter Estimation
- Quadratic optimization: subthreshold
- convex optimization: spike times

7.6. Modeling in vitro data
- how long lasts the effect of a spike?

7.7. Helping Humans

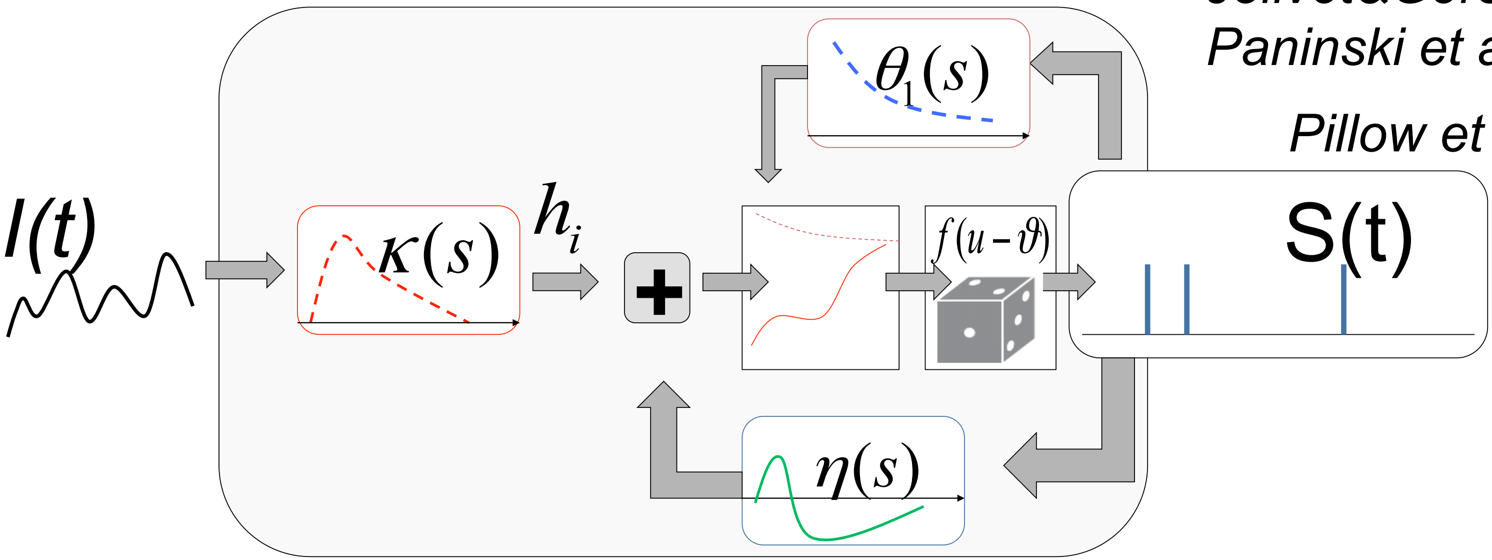
Fitting models to data: so far 'subthreshold'



Neuronal Dynamics – 7.5 Threshold: Predicting spike times

Jolivet&Gerstner, 2005
Paninski et al., 2004

Pillow et al. 2008



potential $u(t) = \int \eta(s) S(t-s) ds + \int_0^{\infty} \kappa(s) I(t-s) ds + u_{rest}$

threshold $\vartheta(t) = \theta_0 + \int \theta_1(s) S(t-s) ds$

firing intensity $\rho(t) = f(u(t) - \vartheta(t))$

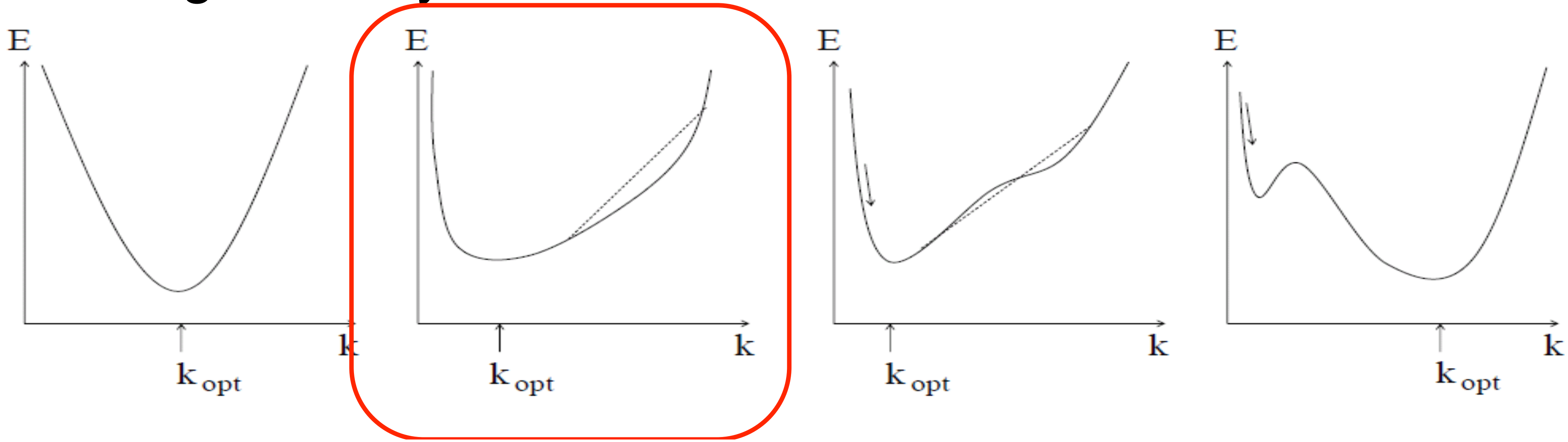
Neuronal Dynamics – 7.5 Generalized Linear Model (GLM)

$$\log L(t^1,\ldots,t^N) = -\int_0^T \rho(t')dt' + \sum_f \log\rho(t^f) = -E$$

potential $u(t) = \int \eta(s) S(t-s)ds + \int_0^\infty \kappa(s) I(t-s)ds + u_{rest}$

threshold $\vartheta(t) = \theta_0 + \int \theta_1(s) S(t-s)ds$

firing intensity $\rho(t) = f(u(t) - \vartheta(t))$



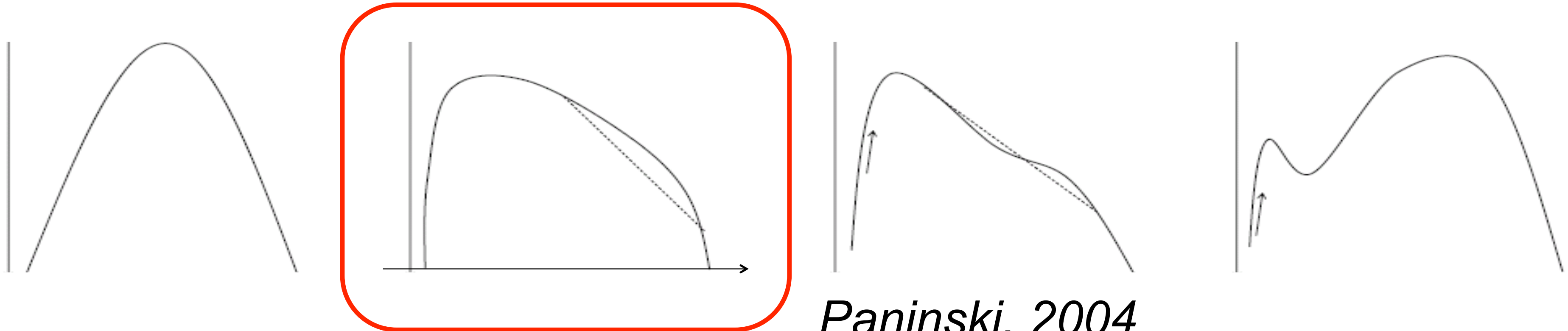
Neuronal Dynamics – 7.5 GLM: concave error function

potential $u(t) = \int \eta(s) S(t-s)ds + \int_0^\infty \kappa(s) I(t-s)ds + u_{rest}$

threshold $\vartheta(t) = \theta_0 + \int \theta_1(s) S(t-s)ds$

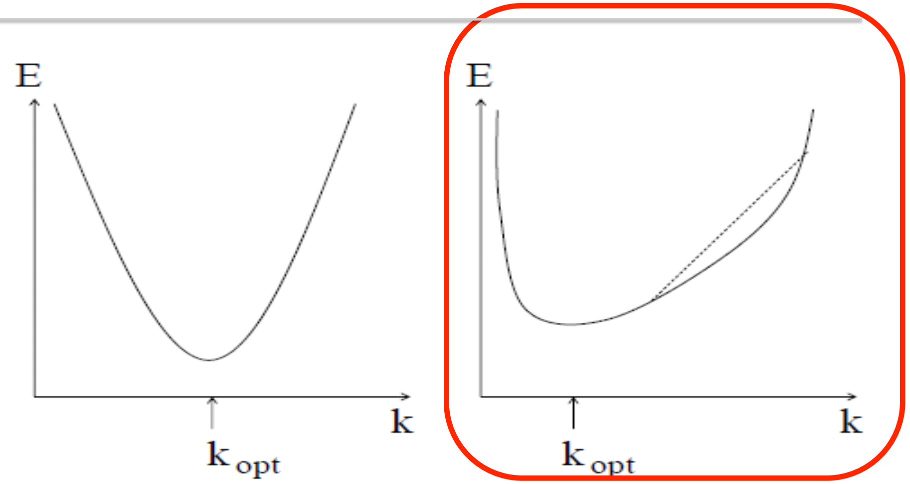
firing intensity $\rho(t) = f(u(t) - \vartheta(t))$

$$\log L(t^1, \ldots, t^N) = -\int_0^T \rho(t')dt' + \sum_f \log \rho(t^f)$$



Paninski, 2004

Neuronal Dynamics – 7.5 quadratic and convex/concave optimization



Voltage/subthreshold

- linear in parameters
  → quadratic error function

Spike times

- nonlinear, but GLM
  → convex error function