Week 7 – part 4 :Generalized Linear Model (GLM)

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Neuronal Dynamics:
Computational Neuroscience of Single Neurons

Week 7 – Optimizing Neuron Models
For Coding and Decoding

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√ **7.1 What is a good neuron model?**
- Models and data

√ **7.2 AdEx model**
- Firing patterns and analysis

√ **7.3 Spike Response Model (SRM)**
- Integral formulation

7.4 Generalized Linear Model
- Adding noise to the SRM

7.5 Parameter Estimation
- Quadratic and convex optimization

7.6. Modeling in vitro data
- how long lasts the effect of a spike?

7.7. Helping Humans

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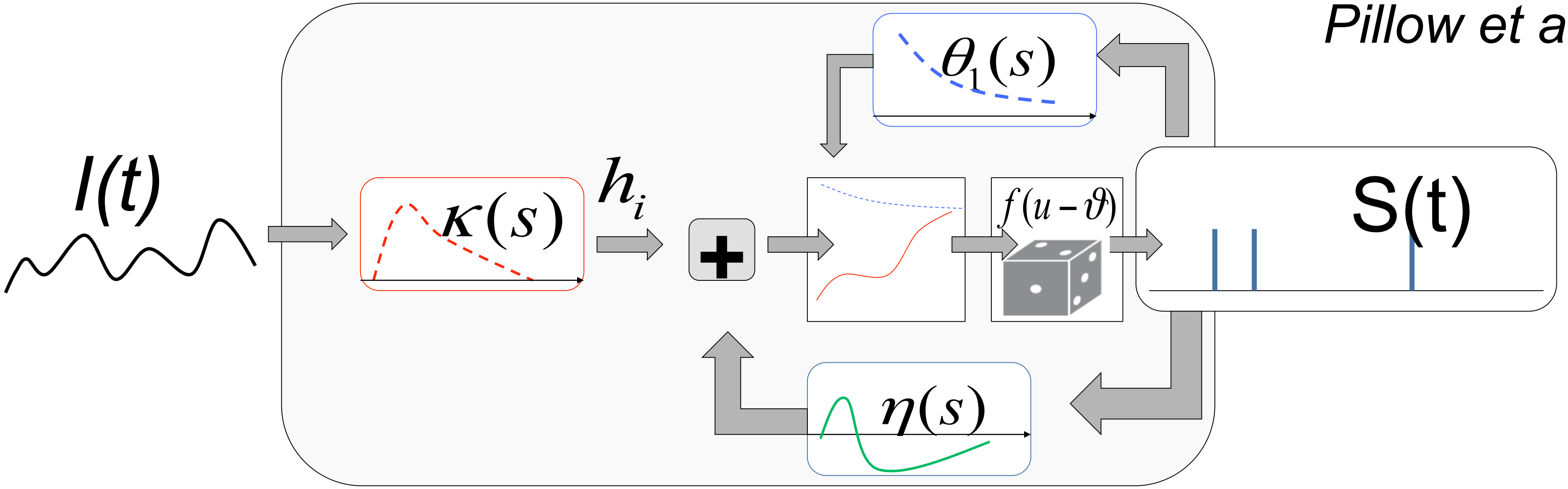
Spike Response Model (SRM)
Generalized Linear Model GLM

Gerstner et al.,
1992,2000
Truccolo et al., 2005
Pillow et al. 2008



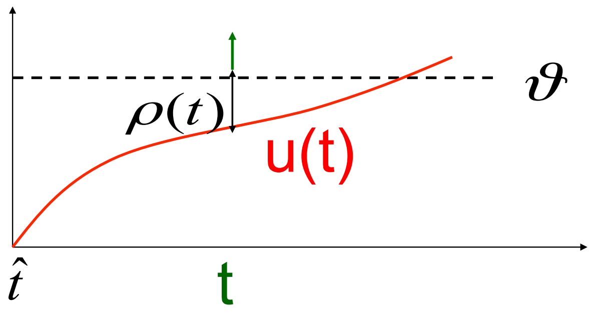
potential $u(t) = \int \underline{\eta(s)} S(t-s) ds + \int_0^\infty \underline{\kappa(s)} I(t-s) ds + u_{rest}$

threshold $\vartheta(t) = \theta_0 + \int \underline{\theta_1(s)} S(t-s) ds$

firing intensity $\rho(t) = f(u(t) - \vartheta(t))$

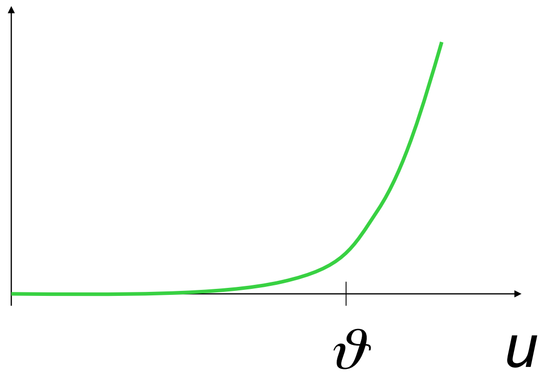
Neuronal Dynamics – review from week 6: Escape noise

escape process



escape rate

$$\rho(t) = f(u(t) - \vartheta)$$



escape rate

$$\rho(t) = \frac{1}{\Delta} \exp(\frac{u(t) - \vartheta}{\Delta})$$

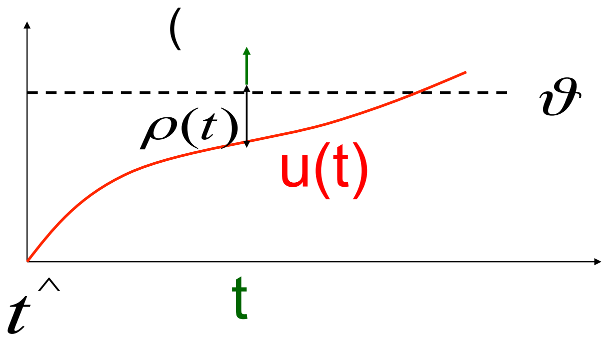
Example: leaky integrate-and-fire model

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

$$if\ spike\ at\ t^f \Rightarrow u\left(t^f + \delta\right) = u_r$$

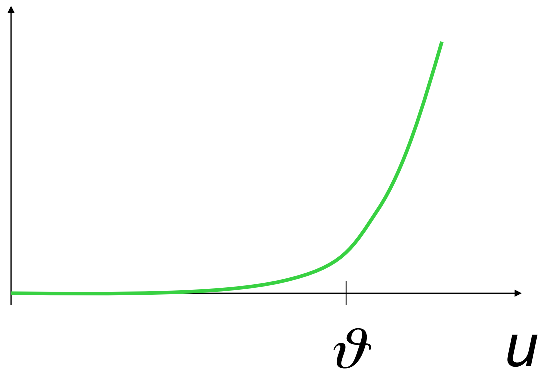
Neuronal Dynamics – review from week 6: Escape noise

escape process



escape rate

$$\rho(t) = f(u(t) - \vartheta(t))$$



Survivor function

$$\frac{d}{dt} S_I(t|\hat{t}) = -\rho(t)\, S_I(t|\hat{t})$$

$$S_I(t|\hat{t}) = \underbrace{\exp(-\int_{\hat{t}}^{t} \rho(t')dt')}$$

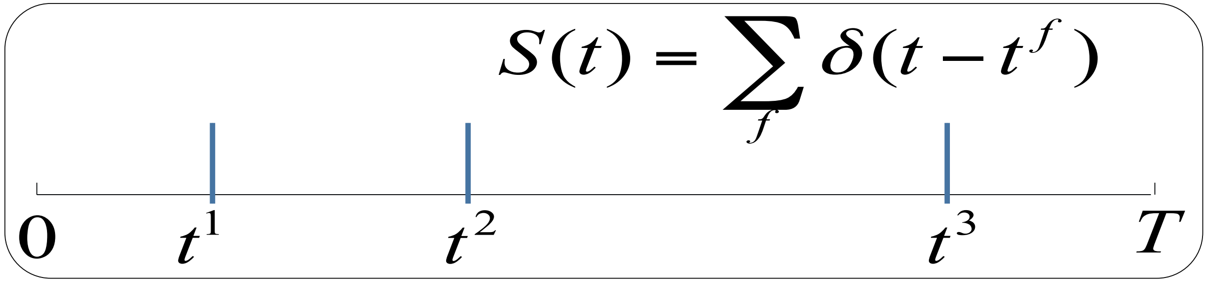
Interval distribution

$$P_I(t|\hat{t}) = \underbrace{\rho(t)}_{\text{escape rate}} \cdot \underbrace{\exp(-\int_{\hat{t}}^{t} \rho(t')dt')}_{\text{Survivor function}}$$

Good choice

$$\rho(t) = f(u(t) - \vartheta(t)) = \rho_0 \exp[\frac{u(t) - \vartheta(t)}{\Delta u}]$$

Neuronal Dynamics – 7.4 Likelihood of a spike train in GLMs



$$S(t) = \sum_f \delta(t - t^f)$$

$$t^1, t^2, \ldots t^N$$

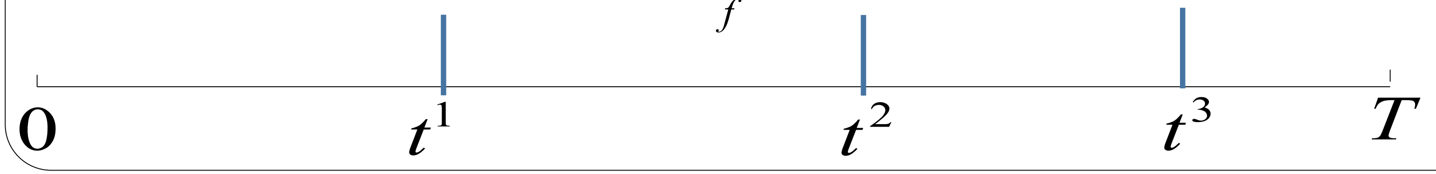
Measured spike train with spike times

Likelihood L that this spike train could have been generated by model?

$$L(t^1, \ldots, t^N) = \exp\left(-\int_0^{t^1} \rho(t')dt'\right)\rho(t^1) \cdot \exp\left(-\int_{t^1}^{t^2} \rho(t')dt'\right) \ldots$$

Neuronal Dynamics – 7.4 Likelihood of a spike train

$$S(t) = \sum_f \delta(t - t^f)$$

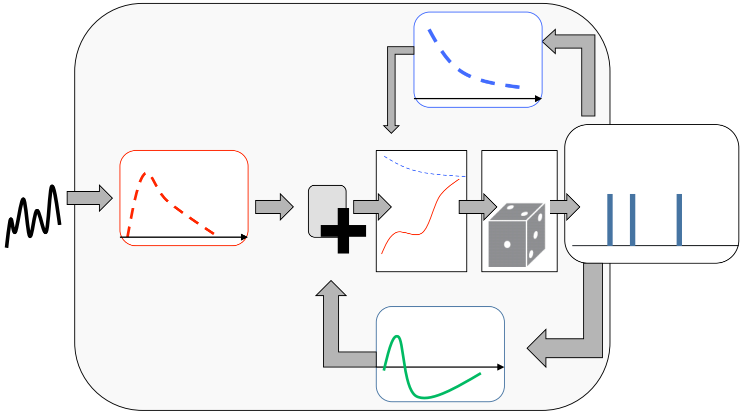


$$L(t^1,...,t^N) = \exp(-\int_0^{t^1} \rho(t')dt')\rho(t^1) \cdot \exp(-\int_{t^1}^{t^2} \rho(t')dt')\rho(t^2)... \cdot \exp(-\int_{t^N}^{T} \rho(t')dt')$$

$$L(t^1,...,t^N) = \exp(-\int_0^{T} \rho(t')dt')\prod_f \rho(t^f)$$

$$\log L(t^1,...,t^N) = -\int_0^{T} \rho(t')dt' + \sum_f \log \rho(t^f)$$

Neuronal Dynamics – 7.4 SRM with escape noise = GLM



-linear filters

-escape rate

→likelihood of observed spike train

→parameter optimization of neuron model