Unit 7: Imperfect Competition I – monopoly

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1 Monopoly without price discrimination

1.1 Basic model

- Model of the consumers is as before:
  - C consumers
  - $U_i(q, m) = B_i(q) + m$
  - w_i endowment of m-good
  - price takers
  - utility maximization problem generates individual demand $x^D(p)$, as before
- Model of the firm is different:
  - 1 firm
  - firm is NOT a price taker
  - Firm knows the aggregate demand function $X^D(p)$ (and thus the inverse aggregate demand $p^D(q)$)
  - The firm's cost function, $c(q)$, satisfies decreasing returns to scale
  - The firm chooses how much to produce to maximize its profits, taking into account how its actions affect market prices
- Monopolist's problem:

- $\max_{q \geq 0} p^D(q)q - c(q)$
- Important: as before, Profits = Revenue - Cost.
- But now price depends on the quantity q supplied by the monopolist!
- FOCs for the monopolist's problem:

$$MR(q) = MC(q)$$
$$q\frac{dp^D}{dq} + p^D = c'$$

- Corner solution if $MR(0) \leq MC(0)$
- SOCs satisfied if marginal revenue decreases with q

• Important: See the graphical depiction of equilibrium in the video lectures

• Intuition: Why does MR tend to decrease with q?

$$dMR = (\overbrace{p}^{+} + \overbrace{qp'}^{-})dq$$

- p term dominates when q small
- qp' term dominates when q large

• Caveat: Possible to construct cases in which $\frac{dMR}{dq}$ not always negative

• Remark: What if there are FCs/SFCs?

- Solve the monopolist's problem in several steps
- Step 1: Compute optimal monopoly profits conditional on positive production: $\Pi^m_{q>0}$
- Step 2: Compute monopoly profits conditional on zero production: $\Pi^m_{q=0}$
- Step 3: Compare the profits at the two cases and select the one with the largest profits:

• Efficiency analysis:

- Compare monopoly outcome to competitive outcome
- q^m = equilibrium quantity in the monopolist case
- q^* = optimal level of production
- Get

$$DWL = \int_{q^m}^{q^*} \left(p^D(q) - c'(q)\right) dq$$

- Why does FWT fail?
  - Compare equilibrium in case of perfect competition vs. monopoly
  - For simplicity, assume every demand and supply function involves an interior solution in equilibrium
  - Competitive market:

$$\begin{array}{ccc} MB_i = & & p = MC_j \\ \uparrow & & \uparrow \\ U\text{max} & & \text{price-taking + profit max} \end{array}$$

$$\implies MB_i = MC_j \implies \text{P.O.}$$

  - Monopoly:

$$\begin{array}{ccc} MB_i = & & p > MR = MC \\ \uparrow & & \uparrow \\ U\text{max} & & \text{monopoly + profit max} \end{array}$$

$$\implies MB_i > MC \implies q^m \text{ inefficiently low}$$

- Distributional analysis:

(see the graph in the video lecture for meaning of symbols)

	Competition	Monopoly	Change
PS	C + D + F	B + C + D	B - F
CS	A + B + E	A	- (B + E)

Get redistribution of SS if concentrated firm ownership

1.2 Example

- Consider a monopolistic market (without price discrimination) with:
  - $X^D(p) = 1000 - p$
  - $c(q) = q^2$
- We get that:
  - $TR(q) = qp^D(q) = q(1000 - q)$
  - $\implies MR(q) = 1000 - 2q$
  - Also, $MC(q) = 2q$
- q^{mon} given by $MR = MC \implies q^{mon} = 250, p^{mon} = 750$

1.3 Sources of monopoly power

- What characteristics of markets lead to monopolistic competition?
- Economies of scale in production:
  - Competitive market forces w/ entry and exit $\implies \#Firms^{LR} \approx \frac{q^*_{LR}}{q^{min}_{ATC}}$
  - When ATC minimized at sufficiently large q, competitive forces can push all firms but one out of market, leading to monopoly
- Network externalities in consumption:
  - Benefit of consumption increases w/ q^{others}
  - E.g. facebook, software, HBO
  - One firm will tend to capture the market, because its value to any consumer increases with the size of its customer base
- Ownership of rare and critical resources:
  - Ex: Suez Canal
  - Ex: rare minerals

- Government assignment of monopoly rights:
  - Arises from political patronage
  - Also used to give firms incentives to make costly infrastructure investments (e.g. roads)
- Patents & copyrights :
  - Innovator's dilemma: large expense required to design a product, but then it can be produced at $MC \approx 0$ + the design is easy to copy
  - E.g. software, movies, medications
  - Patents and copyrights: give monopoly power to innovator for n years to recoup large expense
  - Fundamental trade-off: innovation vs. efficiency in production ex-post

2 Monopoly with price discrimination

2.1 Perfect price discrimination

- Price discrimination
  - Charge different prices to different customers despite identical production costs
  - Ex: Senior citizen discounts at movies
- Basic model of price discrimination:
  - Two types of consumers:
    * n_R Red consumers, each with $p_R^D(q)$ inverse demand function
    * n_B Blue consumers, each with $p_B^D(q)$ inverse demand function
  - No resales
  - Producer perfectly observes consumers' types
  - It charges a *price schedule* $\Pi_c(q)$ to each consumer c

- Each consumer solves $\max_q B(q) - \Pi_c(q)$
- Monopolist has decreasing returns to scale production function w/ no fixed-costs or semi-fixed-costs.
- Monopolist's problem: choose $\Pi_R(\cdot), \Pi_B(\cdot)$ to maximize profits

- Key remark: Monopolist can extract *all* consumer benefit at any q

- Solution to monopolist's problem:
  - By previuos remark, can rewrite monopolist's problem as:

$$\max_{q_R, q_B} n_R B(q_R) + n_B B(q_B) - c(n_R q_R + n_B q_B)$$

  - This problem is identical to the one that characterizes the Pareto optimal allocation.
  - FOCs given by $n_R B'_R = n_R c'$ and $n_B B'_B = n_B c'$
  - Get solution: $q_R^{mon} = q_R^{opt}$ and $q_B^{mon} = q_B^{opt}$

- Remarks
  1. Allocation is Pareto optimal.
  2. PS = SS, CS = 0
  3. Distributional properties of equilibrium depend on distribution of firm ownership
  4. Price schedule is not uniquely defined at optimum (though quantity is)

- How is price discrimination implemented in practice?
  - Model described here is an idealization
  - Perfect price discrimination not possible in practice: too much info required + often illegal
  - However, there are good approximations in practice. Ex: supermarket discount cards

2.2 Quantity discrimination

- Consider a limited form of price discrimination, which entails using very simple price schedules
- Basic model:
  - Simple price schedule for imperfect price discrimination: $(p_1, p_2, \bar{q})$, $p_2 \leq p_1$
  - Consumer can buy up to $\bar{q}$ units at price p_1, additional units sold at p_2
  - Note: when $p_1 = p_2$, this is regular monopoly with no price discrimination
  - Monopolist's cost function: $c(q) = \mu q$
  - C identical consumers, each with inversed demand $p^D(q)$
- Solution to the consumer's problem (see graphs in video lectures for details):
  - Case 1: Price schedule's kink below p^D: buy q^*
  - Case 2: Price schecule above p^D for all q: buy zero
  - Case 3: Price schedule's kink crosses $p^D$and $p_1 < p^D(0)$: $\underline{q}^*$ if $B \geq C$, $\bar{q}^*$ if $B \leq C$
  - Case 4: Price schedule's kink crosses p^D and $p_1 \geq p^D(0)$: 0 if $B \geq C$, $\bar{q}^*$ if $B \leq C$
- Result: With identical consumers, monopolist gets all social surplus and allocation is Pareto optimal
  - Equilibrium price schedule has $p_1 = p^D(0)$, $p_2 = \mu$, and $\bar{q} = {}^{q^{opt}}/_{2}$.
  - Intution: consumers' overpay for initial units, but are willing to do so in order to buy discounted units
- In general, the result does not extend to heterogeneous consumers

2.3 Multi-market discrimination

- Basic model
  - Single firm with centralized production with cost function $c(q)$
  - Firm sells goods in m separate markets
  - Firm allowed to charge different price p_i in each market i, but not to engage in price discrimination within each market
- Firm's problem:

$$\max_{q_1,\ldots,q_m} \sum_i p_i^D(q_i)q_i - c\left(\sum_i q_i\right)$$

- At solution: $MR_i = MC$ in each market i

3 Government policy in monopoly

- What policy instruments can the government use to improve the outcomes generated by monopolistic markets?
- Instrument 1: Promote competition
  - Example: Eliminate government created monopolies
  - Example: Fund research & development in new technologies that could increase competition
  - Instrument inefectual if there are strong 'network effects' in market
- Instrument 2: Regulation
  - Suppose government has full information about market: knows $c(\cdot)$, X^D
  - Then government can compute p^{opt}, q^{opt}
  - Price regulation:
    * Set $p = p^{opt}$

  * Allow monopolist to sell any quantity at price p
- Quantity regulation:
  * Set $q = q^{opt}$
  * Monopolist must produce q, but allowed to charge any unique price
- Remarks:
  1. Regulation replicates allocation of the competitive market equilibrium
  2. Often unfeasible in practice since it requires sufficient information to be able to compute p^{opt}, q^{opt}

- Instrument 3: Subsidization of production
  - Required subsidy per-unit produced: $\sigma = aMB(q^{opt}) - MR(q^{opt})$
  - Monopolist's problem becomes:

$$\max_{q \geq 0} p^D(q)q - (c(q) - \sigma q)$$

  - As before, solution given by FOCs: $MR = MC$
  - Get solution: $p_\sigma^{mon} = p^{opt}, q_\sigma^{mon} = q^{opt}$
  - Remarks:
    1. P.O. possible only if cost of subsidy policy can be financed using lump-sum taxes
    2. This policy has bad distributional properties if firm ownership concentrated in small number of consumers
    3. P.O. requires government to have sufficient information to be able to compute the optimal subsidy

4 Summary

- Markets with monopoly generate very different allocations than those with perfect competition
  - Monopolist without price discrimination: MR = MC, DWL > 0

  - Monopolist with perfect price discrimination: PS = SS, DWL = 0
  - Monopolist with imperfect multi-market price discrimination: DWL > 0

- Feasible policy options provided that the government has sufficient information:

  - Price or quantity regulation
  - Subsidy
  - All these policies restore optimal allocation DWL = 0