Introduction to Aerospace Structures and Materials
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10. Fatigue & durability

10.1 Introduction

The topic of fatigue and durability is an important topic especially for aeronautical structures that are designed to be operated for several decades while assuring the structural integrity. Based upon the discussion in chapter 2, one can imagine that the material properties will change over time due to ageing of the material. Of course this can be accounted for by setting the appropriate allowables based upon the virgin material strength reduced by knock-down factors.

However, although that represents a considerable part of the durability aspect, it will be insufficient for assuring structural integrity. The development of different design philosophies, presented in chapter 9, has proven that structural safety can only be assured with a coherent approach for the design and manufacturing phase, and subsequent operational life.

An important aspect in this sense is the occurrence of fatigue and fatigue damage in structures that are repetitively loaded in service under various loading and environmental conditions. It will be explained in this chapter that fatigue cannot be treated independent of environmental aspects, part of the durability assessment. Therefore, fatigue and durability are discussed in relation to each other in this chapter.

There are some remarks to be made here before stepping into the details. First, fatigue as a damage phenomenon has been discovered primarily in metallic structures (steel in train applications, aluminium in aircraft applications). What has been understood is that from the decades of experience and research is that repetitive (tensile) loading may lead to initiation and propagation of cracks that at some point in time may lead to component failure. It has also been understood how this aspect should be treated in design, which has led to the development of methods for designing against fatigue.
What seems to be not well understood is that the relation between repetitive or cyclic loading and initiation or formation of damage applies to all engineering materials and material categories, but that the nature of damage and thus the relevant cyclic load cases may be different from one material category to another. This will be further explained in section 10.4.6.

Another aspect requires more awareness is that designing against fatigue is not an additional task one should perform in the detail design phase. Designing against fatigue follows from the correct mind set of an engineer and should be incorporated already in the early conceptual and preliminary design. If fatigue has not been considered in the early design phases, the design work in the detail design phase will be merely repairing a fatigue sensitive structure.

10.2 Stress and strain concentrations

10.2.1 Definition

Following from the definitions provided in chapter 1, the stress in a material or component is basically calculated by dividing the applied force by the area. Or in equation form:

\[ \text{Stress} = \frac{\text{Applied Force}}{\text{Area}} \]
This is considered correct if the material is homogeneous and in undisturbed in shape or geometry. This concept is illustrated in Figure 10.1. Independent on where a cross-section is considered, the stresses will be equal and evenly distributed through the material. These stresses can be calculated with equation (10.1).

However, if the geometry or the shape is changed, or if a cut, damage or crack is created in the material, then this forms a disturbance for the stress field. The load that has to be transferred through the material, illustrated in Figure 10.2 with stress lines, has to bypass the cut or damage.

Assuming that equation (10.1) is still valid, one should at the location of the disturbance calculate the stress by dividing the applied load $F$ by the reduced cross section. Because the cross-sectional area is reduced while the load remains the same, this implies that the stress at the location of the disturbance is higher than in the undisturbed stress field. This higher stress is generally denoted as the nominal stress and is defined by

$$
\sigma_{\text{nom}} = \frac{F}{A^*} = \frac{A}{A^*} \frac{F}{A} = \frac{A}{A^*} \sigma_{\text{gross}}
$$

(10.2)

This would suggest that if the failure of a disturbed or cracked component has to be predicted, one may assume that once the nominal stress $\sigma_{\text{nom}}$ equals ultimate strength of the material, failure would occur. However, before this equilibrium has been reached, the material or component will fail.

This phenomenon is related to the fact that a concentration of stress is present at the tip of the notch. This is illustrated with the higher density of stress lines at the notch tip in Figure 10.2. The undisturbed and evenly distributed stress field has to bypass
the notch, which at the location of the end of the notch creates a high concentration of stress. This phenomenon is indicated with the term stress concentration.

To evaluate the stress concentration, a stress concentration factor $K_t$ has been defined

$$K_t = \frac{\sigma_{\text{peak}}}{\sigma_{\text{nom}}}$$

(10.3)

Here the peak stress is the highest stress right at the edge or tip of the notch and the nominal stress $\sigma_{\text{nom}}$ is calculated with equation (10.2).

Consider a flat panel with width $W$ containing a circular hole with diameter $D$, see Figure 10.3, the nominal stress can be calculated with

$$\sigma = \frac{W}{W - D} \sigma_{\text{applied}}$$

(10.4)

The stress concentration factor can then be calculated with equation (10.3). The only question at this point will be whether the peak stress is known. Because people in the past did not have the availability of dedicated software that can be used to calculate stresses at specific locations for any geometry under load, like with for example Finite Element Software, theoretical solutions were mathematically derived. These theoretical solutions were derived for the case of notches in sheets with infinite dimensions, because mathematically the boundary constraints for this case are exactly known.

For the finite geometries, subsequently correction factors were determined based upon calculations or experimental data. These solutions were combined in handbooks, like for example Peterson’s Stress concentration factor (Pilkey, W.D., 1997).
A well-known general solution for an elliptical hold in an infinite sheet illustrated in Figure 10.4 is given by:

\[ K_t = \frac{\sigma_{peak}}{\sigma_{nom}} = 1 + \frac{2a}{b} = 1 + 2\sqrt{\frac{a}{r}} \]  

(10.5)

This means that the stress concentration factor for a circular hole in an infinite sheet is equal to \( K_t = 3 \). This value is indifferent from the hole diameter. The relation between the peak stress at the notch and the nominal stress in the remaining cross-section section remains 3, independent what hole diameter is considered.

An important remark has to be made. When introducing the stress concentration factor \( K_t \) and the peak stress at the notch edge, it was argued that the component would fail before the nominal stress \( \sigma_{nom} \) would have reached the ultimate strength of the material.

Implicitly, this explanation and subsequent derivation may lead to the assumption that failure would thus occur when the peak stress \( \sigma_{peak} \) reaches the ultimate strength. This is correct for materials that remain (approximately) linear elastic until failure, like for example fibre reinforced polymer composites.

However, metallic materials usually plastically deform beyond the yield strength. In that case the behaviour will be different. This is illustrated in Figure 10.5. The stiffness of material reduces significantly beyond yielding (evident from the tangent slope
compared to the linear elastic slope, represented by the Young's modulus E). As a result of this local stiffness reduction, stress will be redistributed to the remaining material.

![Diagram showing stress concentration](10-5.jpg)

Figure 10.5

The example given on the next page, illustrates that linear elastic materials have significant higher notch sensitivities than ductile materials. As a result, the failure strength of a notched component made of linear elastic materials is usually lower than that of equivalent elastic-plastic materials.

On the other hand, as will be discussed later in this chapter, composites are usually less sensitive to fatigue loads. In other words, comparing aerospace aluminium alloys with carbon fibre composites, one may conclude that stress concentrations are detrimental for static loads on composites and detrimental for fatigue loads in aluminium alloys.

10.2.2 Saint Venant’s principle

An important principle for structural analysis is the principle of Saint Venant. This principle states that the effect of a local disturbance, due to for example a notch, remains limited to the direct neighbourhood of the location of that notch, see Figure 10.6.
Example: Effect of plasticity on $K_t$

Consider a straight sample containing a hole, as illustrated in Figure 10.5. If the material behaviour would be linear elastic until failure, like for example carbon fibre composite, this sample would fail when $\sigma_{peak} = \sigma_{ult}$, which means that the gross strength of failure is equal to

$$\sigma_{applied} = \frac{W}{W-D}\sigma_{nom} = \frac{\sigma_{ult}}{K_t}$$

For a sheet with infinite dimensions, this means that the failure strength is equal to $\sigma_{ult}/3$. For isotropic carbon fibre composite the failure strength can be about 500 MPa, which means that failure of the sample would occur at 166 MPa.

If the material behaviour would be elastic-plastic like for example aluminium

$$\sigma_{applied} = \frac{W}{W-D}\sigma_{nom} = 0.9\sigma_{ult}$$

For a sheet with infinite dimensions, this means that the failure strength is equal to about 0.9 $\sigma_{ult}$. With the failure strength of aluminium 2024-T3 of about 450 MPa, this implies failure would occur at about 405 MPa.
Do consider however that the density of aluminium 2024-T3 is about 2.7 g/cm$^3$ (providing a specific failure strength of 150 kN*m/kg), while the carbon fibre composite would be about 1.5 g/cm$^3$ (thus a specific strength of 110 kN*m/kg).

The benefit of this principle is that the stress analysis of a loaded structure can be divided into a

- global stress analysis (considering the structure as a whole)
- local stress analysis of all details (containing notches)

This approach is justified by the Saint Venant’s principle; a structure faces the same stresses due to applied loading indifferent of the presence of a notch at a certain location, except for the location of the notch itself.

10.2.3 Stress distribution around a hole

For the sake of simplicity, the stress concentration factor $K_f$ has been introduced while limiting the discussion to the stress in the net section (nominal stress) and the stress at the notch edge (peak stress). However, it is obvious that the disturbance in stress created by the presence of the notch is somewhat more complex. The peak stress described by $K_f$ is only at the location in the net section. Away from the hole edge or along the contour of the notch, the stresses reduce and may even become negative.
This is illustrated in Figure 10.7. In this figure a peak stress of 3S is present at both sides of the hole, while a compressive stress is present at the locations given by $\phi = \pm \pi/2$. This compressive stress may be understood when considering the stress lines similar to the ones illustrated in Figure 10.2. These stresses may be decomposed in stresses in horizontal and vertical direction, where the horizontal component just above and below the hole induce a compressive stress.

Another aspect that can be observed in Figure 10.7 is that the stress drops quickly moving away from the hole, while the drop along the contour is significantly less. This implies there is a certain area along the notch contour that may be considered relatively high stressed (in the order of $0.9 \sigma$), where potentially fatigue cracks may initiate.

10.2.4 Superposition principle

According to the definition given in section 10.2.1, the stress concentration factor $K_t$ is linearly dependent on the stress. Because stresses may be superimposed, the stress concentration factor $K_t$ coming from a complex set of stresses may be derived by addressing the different stress systems separately and subsequently superimposing the (peak) stresses at the notch.
This is illustrated for a bi-axially loaded infinite sheet containing a hole in Figure 10.8. To calculate the stress concentration factor $K_r$ both stress systems $\sigma_1$ and $\sigma_2$ may be considered separately. For the stresses in direction $\sigma_1$ a peak stress of $3 \sigma_1$ may be observed at the hole edge as indicated in the figure. However, looking at the stress $\sigma_2$ then a compressive stress equal to $-\sigma_2$ is present at that location (see Figure 10.7). The actual peak stress at the considered location is therefore less than $3\sigma_1$.

In other words, the stress concentration factor for uni-axially loaded sheet containing a hole is higher than the same sheet bi-axially loaded in tension. If the sheet would be bi-axially loaded by a tensile stress $\sigma_1$ and a compressive stress $\sigma_2$ (thus opposite to the sign in Figure 10.8), the stress concentration factor would be higher than the uni-axially loaded case.

![Figure 10.8](10-8.jpg)

*Figure 10.8*
10.3 Reinforcement or weakening?

10.3.1 Train axles

The significance of the theory on stress concentrations can be illustrated with the example of train axles. Train axles are heavily loaded components that due to the high rotational speed are repetitively loaded in (reversed) bending. One could consider reducing the load by reinforcing the axle. Reinforcement implies that the axle becomes stronger, but after quite an amount of train axle failure in the 19th century people became aware that reinforcing may imply a weakening.

Example: Versailles railway accident

On the 5 October 1842 a disastrous accident occurred near Versailles. This is the first major train accident reported in history. The accident occurred due to a fatigue failure of a locomotive front axle, causing collision and derailing of the locomotives. In the subsequent fire about 60 people died.

The investigation into the cause of this accident was performed by Rankine. He recognized distinctive characteristics of fatigue failure and the danger of stress concentrations that locally raise the stress. The Versailles accident and the subsequent railroad accidents led to intensive investigations of train axle, coupling and rails failures due to fatigue. Almost within a decade after 1842 over hundreds of broken axles had been reported.

It is important to realise the significance of this accident and the magnitude of fatalities. About the same number of fatalities occurred in the two Comet accidents in 1954 a century later, see chapter 9. These two accidents still are
considered a major milestone in the history of aviation and the initiator of the investigations into the fatigue phenomena.

This phenomenon can be illustrated with Figure 10.10. The evenly distributed stress in the section of the axle with diameter \( d \) has to change into a lower evenly distributed stress in the section of the axle with diameter \( D \). This disturbance of the homogeneous stress field in the radius of the thickness step causes a concentration of stress with a peak stress at the radius contour.

The radius becomes therefore important; a small radius induces a high stress concentration, while a large radius reduces the stress concentration significantly. Often this implies some engineering, because very large radii are not always possible.

<table>
<thead>
<tr>
<th>( r/D )</th>
<th>0.04</th>
<th>0.1</th>
<th>0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_t )</td>
<td>3</td>
<td>2</td>
<td>1.5</td>
</tr>
</tbody>
</table>

\[ K_t \]
10.3.2 Repairs

The example discussed in the previous section should be seriously considered, because it relates to a general problem. If a structure in general is reinforced with additional material or structure, the influence of the geometry may induce unwanted concentration of stress.

This may also be the case when for example an aircraft structure is reinforced with doublers or when such structure is repaired with patch repairs. The thickness step related to the doubler or patch implies a location for stress concentrations. This may be even more important when considering the higher stiffness of the reinforced area.

This is illustrated with examples in Figure 10.11. Especially the illustration in this figure highlights using similar stress lines as in Figure 10.2, that the stiffer area of the patch repair attracts more load than originally would be present at that location in the structure. The patch may successfully reinforce the damage it originally repaired, but the higher stress with the thickness step induced stress concentration may shift the critical areas to the edge of the patch repair.

A solution to prevent a failure at this location may be to taper the edge, as illustrated with the bonded Glare repairs on the C-5 Galaxy (right hand side of Figure 10.11).
10.4 Fatigue

10.4.1 Definition

It has been mentioned in the introduction to this chapter that fatigue as phenomenon has been well understood for metallic materials and structure, see Schijve (2001). However, despite this knowledge and understanding, there seems to be misperception with respect to the phenomenon of fatigue when considering different material types, like for example fibre reinforced composites. The misperception is illustrated with the statement often put forward, that composites do not suffer from fatigue.

If that statement is made, it may refer to two cases:

- When cyclic loading composites in tension, similarly to common loading of metallic materials, composites hardly show evidence of damage initiation and growth, where fatigue cracks may initiate and propagate in metals.
When designing a composite structure, addressing the static strength requirements usually is sufficient to cover fatigue.

The first aspect may be true, but in return there may be repetitive load cases that do not cause any fatigue damage in metallic materials, that do initiate and propagate damage in composites. As example, certain compressive load cycles severely damage composite structures when applied many times, which do not cause any fatigue damage in metallic materials.

The second aspect, relates partly to the discussion of the stress concentration factor; linear elastic material is very sensitive to notches, which is not the case for metallic materials. Therefore metallic materials can be more efficiently loaded up to their static strength. In addition, reduction due to environmental impact (see chapter 2), further restricts the static allowables of a composite design. When these restrictions are accounted for, the corresponding load spectra do not cause fatigue issues in the structure.

However, the statement and the above mentioned considerations may never refer to the conclusion that composites do not face damage phenomena under any repetitive or cyclic load spectra. In general, it can be stated that all engineering materials in one way or another suffer from fatigue. Only the phenomena appear to be different leading to different failure mechanics under different loading conditions.

To do justice to the generic aspect of fatigue, fatigue can therefore be defined as:

*Damage phenomenon induced by a large number of load cycles below the ultimate strength of material or structure causing permanent deterioration of material or structure resulting in a reduction in load bearing capability.*

For aeronautical structures, the load cycles mentioned in this definition are often related to number of flights, ranging from approximately constant amplitude (fuselage pressurization every flight) to arbitrary spectrum loading (wing loading during take-off, landing, and turbulence). This is illustrated with an arbitrary flight load spectrum shown in Figure 10.12. In this example the maximum load in the whole spectrum is called nominal or limit load. This is the load case that is considered to occur only once in a lifetime. At this load failure may not occur. Ultimate load, used for strength justification is usually calculated as 1.5 times the nominal or limit load.
10.4.2 Fatigue assessment

It was noticed by the researchers investigating fatigue after the railroad failures that fatigue appears to be predominantly influenced by the amplitude of the load cycles. Wohler, who performed an extensive research at the time on fatigue loading of train axles, concluded that the

"Material can be induced to fail by many repetitions of stresses, all of which are lower than the static strength. The stress amplitudes are decisive for the destruction of the cohesion of the material. The maximum stress is of influence only in so far as the higher it is, the lower are the stress amplitudes which lead to failure."

For this reason, the fatigue characteristics are traditionally represented by so called Wohler curves, or S-N curves, see for example Figure 10.13.
One should carefully pay attention to the S-N curve as illustrated in Figure 10.13. In this figure, three curves are given for three different mean stress levels. It is illustrated that the lower the mean stress $\sigma_{\text{mean}}$ the higher the curve. This relates to the conclusion of Wohler; the stress amplitude is decisive and thus plotted on the vertical axis, but there is an effect of maximum stress. The lower the mean stress for given stress amplitude, the lower the maximum stress $\sigma_{\text{max}}$. This will increase the fatigue life (equivalent to shifting the curve to the right), which is equivalent to shifting the curve upward.

To understand this, one has to consider the stress cycle. The relation between the amplitude stress $\sigma_a$ and the mean stress $\sigma_{\text{mean}}$ on the one hand, and the minimum stress $\sigma_{\text{min}}$ and maximum stress $\sigma_{\text{max}}$ on the other hand is given by:

$$\sigma_a = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2}; \quad \sigma_{\text{mean}} = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} \quad (10.6)$$

Note that the assessment of fatigue using S-N curves is solely based on evaluating fatigue under constant amplitude loading, i.e. all load cycles have the same mean stress and stress amplitude. As earlier mentioned, except for pressurization of fuselages all flight load spectra are random of nature and thus to be considered as variable amplitude loading.

In general, two asymptotes can be identified in the Wohler curve

- Upper asymptote, related to the maximum stress reaching the ultimate strength
• Lower asymptote, defined as fatigue limit

The lower asymptote is an important parameter, when evaluating fatigue for specific materials or structures. Especially, when high cycle fatigue is considered, i.e. fatigue with a very large number of cycles (order $10^5$ or higher). The fatigue limit is defined as the stress amplitude (usually given for a mean stress of $\sigma_{\text{mean}} = 0$), below which fatigue failure do not occur. This does not mean that nucleation of microscopic fatigue damage does not occur, but that even if such damage does occur that it will never propagate to macroscopic lengths causing failure.

One way to design against fatigue is to limit the operational load spectra to stress amplitudes below the fatigue limit. This approach formed the basis for the Safe-Life concept discussed in chapter 9.

10.4.3 Variable amplitude loading

The assessment of flight load spectra, or variable amplitude fatigue loading, is in general performed using the constant amplitude data as represented in Wohler curves, see Figure 10.13. The most well-known method widely applied to do approximate the variable amplitude behaviour (and also well known to be very inaccurate) is the Miner rule.

This rule is a linear damage accumulation rule that sums the damage fractions of each individual load cycle in the load spectrum until the total damage fraction reaches the value 1. The material or structure is then considered failed.

The damage fraction of for example the $i^{\text{th}}$ load cycle in the spectrum is calculated with

$$D_i = \frac{1}{N_i} \quad (10.7)$$

where $N_i$ is the fatigue life determined with the constant amplitude S-N data, see Figure 10.13. If the load spectrum contains more of the same load cycle, this relation is written as:

$$D_i = \frac{n_i}{N_i} \quad (10.8)$$

Failure of the component then will be assumed to occur at:
Again, one should note that this simple rule ignores several aspects that play a role in the fatigue phenomenon. As a consequence, this rule is often inaccurate. The engineering approach to account for that is to calibrate the rule to specific materials and load spectra. This calibration, or more correct empirical correction, may lead to values smaller or larger than 1.

\[
\sum_{i} \frac{n_i}{N_i} = 1 \quad (10.9)
\]

10.4.4 Notched materials and structures

The above discussion has been presented assuming there are no notches present in the material or structure. The fatigue limit mentioned in section 10.4.2 is then considered to be a material parameter. However, following from the discussion of stress concentrations at the beginning of this chapter, it is obvious that fatigue may occur sooner at a notch where a peak stress is present.

The effect of a stress concentration, described by the factor \( K_t \), on the fatigue behaviour can be illustrated with the S-N curve as given in Figure 10.14. The upper asymptote, related to \( \sigma_{ult} \), is not affected for metallic materials, because of the plasticity redistributing the stress, see Figure 10.5. Only a small, often negligible, reduction will occur. Be aware that this is not the case for linear elastic materials, like composite materials. For these materials the upper limit will reduce by a factor \( K_t \).

For both metallic and composite materials, there is a significant effect on the fatigue limit. Because of the higher peak stress at the notch, fatigue damage may initiate sooner than would be expected based on the undisturbed stress field data. In general, the fatigue limit reduces by a factor \( K_t \) for notched structures.

For small notches and large stress concentration factors \( K_t \), this reduction may be less severe for metallic materials. Again the plasticity provides an advantage over linear elastic materials. High values for \( K_t \) implies theoretically high values for the peak stress. However, similar to the discussion on Figure 10.5, the peak stress is levelled off beyond the yield strength of the material. As a consequence, the real peak stress is lower.

Following from the definition given by equation (10.3), this implies a lower stress concentration. The actual stress concentration due to plasticity is therefore described by a notch factor \( K_f \), which is lower than the stress concentration factor \( K_t \). Again,
note that this only the case for ductile (thus elastic-plastic) materials and not for linear elastic materials, like composites. Composites are considered to be very notch sensitive.

Figure 10.14

10.4.5 Designing against fatigue

The fatigue behaviour represented by the S-N curves is considered up until failure. It was already mentioned that below the fatigue limit, microscopic damage may occur but will not lead to failure. The fatigue life governed by the S-N curve, consists for 80-90% of formation of a fatigue damage of about few millimetres (nucleation of microscopic damage up to macroscopic lengths). Only a few percent of the life consist of a crack that propagates to a larger length causing final failure.

The Safe-Life concept implies designing against fatigue using fatigue life data (S-N curves). To justify such design, large safety factors are required with respect to available fatigue data. The experimental life (proven by tests) must be about 3 to 4 times the life that the aircraft is certified for. This is related to the fact that the experimental data shows considerable scatter, especially at lower stress amplitude cycles.

Although the S-N curves are presented like in Figure 10.13 and Figure 10.14, one has to be aware that these curves represent average or minimum data. Performing
individual tests near the fatigue limit for example, may result in fatigue lives that deviate by a factor 10 from the curve.

Because of the large margin of safety applied using the Safe-Life concept, the thickness of fatigue critical parts is often significant larger than required for static strength. Especially, for metallic materials this is the case.

Designing against fatigue then aims to increase fatigue lives or to increase allowable fatigue stresses for given operational life. This implies lowering stress concentrations and avoidance of damage initiators in the design, but also selection of materials that are less fatigue sensitive (higher fatigue limits). However, this can be achieved also by applying a damage tolerance philosophy rather than a Safe-Life concept.

10.4.6 Some fatigue characteristics

It has been mentioned a few times in this chapter that fatigue is a generic damage formation phenomenon related to cyclic loading. The appearance of the damage may be different from one material type to another. For example, in metallic materials, the cyclic (tensile) loading may initiate and propagate cracks, whereas in fibre reinforced composites, delaminations and transverse shear cracks may occur.

The type of loading for formation of these damage types is often different. As said, cracks in metals are often driven by tensile load cycles applied in plane of the material. The delaminations in composites are often induced by cyclic compression, shear or bending loads.

Looking at these materials on a microscopic level, one may see some qualitative commonalities that play a role in the nucleation of damage. In metals and alloys, often the inclusions in the material cause stress concentrations at microscopic level, because the mechanical properties of inclusion and surrounding material are different, see Figure 10.15. However, this difference in mechanical properties is also present for longitudinal fibres and matrix in composites. Qualitatively, both cases imply stress concentrations that may case tensile cracks in metals and transverse shear cracks and delaminations in composites.
Despite the qualitative microscopic correlation in Figure 10.15, fatigue appears to be different in metals and composites. The fatigue damage of both cases in this figure are nucleated by different loads on a macroscopic level; metal are most sensitive to tension-tension loading, while composites are most sensitive to compression-tension or compression-bending loading. Some illustrations of typical fatigue failure mechanisms on a macroscopic level are illustrated in Figure 10.16.

Figure 10.15
Qualitative comparison between metal alloys containing inclusions (left) and fibre reinforced composites (right); in both cases stress concentrations are present at micro level. Derivative from left: Antiproton (2017), CC0 and right: MT Aerospace AG, (2006), CC-BY-SA3.0.

Figure 10.16
10.5 Damage tolerance

10.5.1 Definition

There are different definitions for damage tolerance. Following from the discussion in chapter 9, damage tolerance represent a design philosophy where structural integrity is assured even in presence of potential damages by a combination of built-in design features and inspection and maintenance procedures. This definition is very relevant for the aeronautical field of structural design.

However, sometimes a more strict or limited definition is used that relates to the former definition, but omits the aspect of possible inspection and maintenance. This definition relates to the material’s response to potential damage cases. Traditionally, this definition included fatigue cracks and environmental damages, but also impact damages. Interestingly enough, with the introduction of composites in primary aircraft structures, people seem to further limit this definition to impact damages alone. This is probably related to the fact that composites are considered not to suffer from fatigue, which has been discussed before.

However, it is recommended to consider all relevant damage scenarios even when addressing damage tolerance as a material aspect only. This aspect is considered in the definition of residual strength, which considers the remaining load bearing capability of material or structure in presence of potential damage. In the last section of this chapter, this aspect will be briefly addressed for a material or structure containing a fatigue crack.

10.5.1 Limitations of stress concentration concept

The stress concentration factor $K_t$ captures the effect of disturbance in the stress field due to the presence of a notch. However, when addressing the residual strength (or damage tolerance) of a material or structure, there is a limitation to the concept. This can be illustrated with the description of equation (10.5). The stress concentration is directly related not only to the size of notch, but also the radius at the critical location.
Consider that a fatigue crack has initiated at the edge of a hole and propagated a few millimetres, he question what the residual strength is cannot be answered using the stress concentration factor. Because a crack has a sharp tip, which may be considered to have a infinitely small radius. With equation (10.5) this implies that the stress concentration factor becomes infinite, indifferent from the size of the crack. However, it does not require a lot of fantasy to understand that a structure with a small crack is capable to carry a higher load than the same structure containing a long crack.

In other words, to assess the residual strength another parameter is needed than the stress concentration factor \( K_t \). What can be learned from equation (10.5) is that apparently, a crack with a sharp tip induces a singularity in the stress field right at the tip. To mathematically derive a relevant parameter describing this stress singularity first a crack is considered in a sheet with infinite dimensions. This case is considered for similar reasons as explained for the stress concentration factor: the boundary conditions are exactly known.

The stress field including the singularity can be mathematically derived for this case. Although not necessary to reproduce in this course, for illustration the stress field is given here

\[
\begin{align*}
\sigma_x &= \frac{S\sqrt{\pi a}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} (1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2}) - S \\
\sigma_y &= \frac{S\sqrt{\pi a}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} (1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2}) \\
\tau_{xy} &= \frac{S\sqrt{\pi a}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \sin \frac{3\theta}{2}
\end{align*}
\]  

(10.10)

It can be observed that these equations have a component in the equation that describes the location measured with respect to the crack tip. The other part in the equations has a similar appearance for each equation. It is this part of the equation that is taken as parameter, indicated by \( K \).

\[ K = S\sqrt{\pi a} \]  

(10.11)

This parameter is called the stress intensity factor (not to be confused with stress concentration factor). In this relation the crack length is evidently related to the dimensions of the crack (which was no longer described with \( K_t \)). In addition, the applied stress levels have an influence on the stress intensity.
When considering cracks, different opening modes can be identified, see for example Figure 10.17. These opening modes imply different behaviour. Obviously, fatigue cracks in metallic materials and structures under cyclic tensile loading predominantly focus on the tensile mode I.

The stress intensity factor $K$ for a crack opening in mode I, is indicated with $K_I$ (subscript denotes the opening mode). The residual strength of a material containing a crack is evidently then related to the critical value of this $K_I$. This critical parameter, $K_{IC}$, is called the fracture toughness, previously discussed in chapter 1. This fracture toughness is considered a material specific material. Materials with high fracture toughness are clearly capable to sustain high loads in presence of cracks, or considerable loads in presence of very long cracks, see equation (10.11).

Thus, if the fracture toughness is given by:

$$K_{IC} = S_{crit} \sqrt{\pi a_{crit}}$$

(10.12)

The sensitivity of materials to cracks can be illustrated with Table 10.2, assuming that the critical stress is related to the material’s yield strength by $\sigma_{crit} = \sigma_{0.2}/2$. From the comparison it can be observed that despite steel having the highest fracture toughness of these four materials, the critical crack length is the smallest. Evidently, the influence of ductility plays an important role here. Ductile alloys, having low yield strength, are usually less sensitive for cracks.
<table>
<thead>
<tr>
<th>Alloy</th>
<th>$\sigma_{0.2}$ [MPa]</th>
<th>$\sigma_{\text{crit}}$ [MPa]</th>
<th>$K_{\text{ic}}$ [MPa m$^{1/2}$]</th>
<th>$a_{\text{crit}}$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2024-T3</td>
<td>360</td>
<td>180</td>
<td>40</td>
<td>15.7</td>
</tr>
<tr>
<td>7075-T6</td>
<td>470</td>
<td>235</td>
<td>27</td>
<td>4.2</td>
</tr>
<tr>
<td>Ti-6Al-4V</td>
<td>1020</td>
<td>510</td>
<td>50</td>
<td>3.1</td>
</tr>
<tr>
<td>4340 steel</td>
<td>1660</td>
<td>830</td>
<td>58</td>
<td>1.55</td>
</tr>
</tbody>
</table>
This book provides an introduction to the discipline of aerospace structures and materials. It is the first book to date that includes all relevant aspects of this discipline within a single monologue. These aspects range from materials, manufacturing and processing techniques, to structures, design principles and structural performance, including aspects like durability and safety. With the purpose of introducing students into the basics of the entire discipline, the book presents the subjects broadly and loosely connected, adopting either a formal description or an informal walk around type of presentation. A key lessons conveyed within this book is the interplay between the exact science and engineering topics, like solid material physics and structural analysis, and the soft topics that are not easily captured by equations and formulas. Safety, manufacturability, availability and costing are some of these topics that are presented in this book to explain decisions and design solutions within this discipline.