Week 4 – MathDetour 3: Stability of fixed points

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Neuronal Dynamics:
Computational Neuroscience of Single Neurons

Week 4 – Reducing detail:
Two-dimensional neuron models

Wulfram Gerstner

EPFL, Lausanne, Switzerland

√ 4.1 **From Hodgkin-Huxley to 2D**

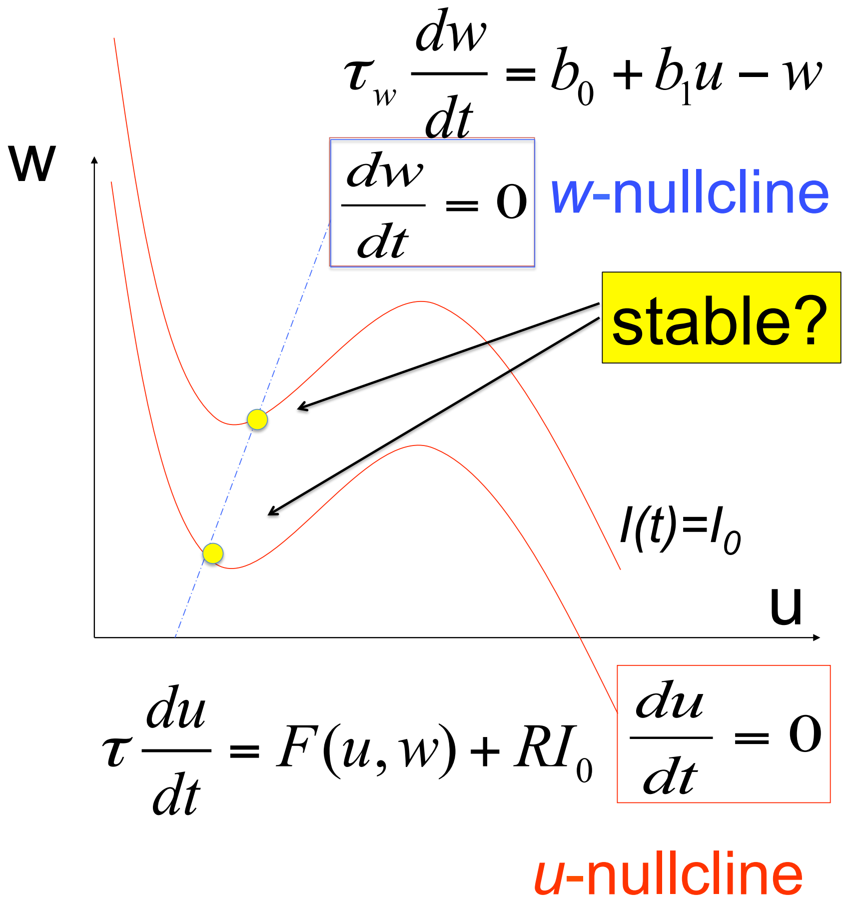
√ 4.2 **Phase Plane Analysis**
- Role of nullcline

4.3 **Analysis of a 2D Neuron Model**
- MathDetour 3: Stability of fixed points

4.4 **Type I and II Neuron Models**
- where is the firing threshold?
- separation of time scales

4.5. **Nonlinear Integrate-and-fire**
- from two to one dimension

Neuronal Dynamics – Detour 4.3 : Stability of fixed points.



Neuronal Dynamics – 4.3 Detour. Stability of fixed points

2-dimensional equation

stimulus

$$\tau \frac{du}{dt} = F(u,w) + RI_0$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

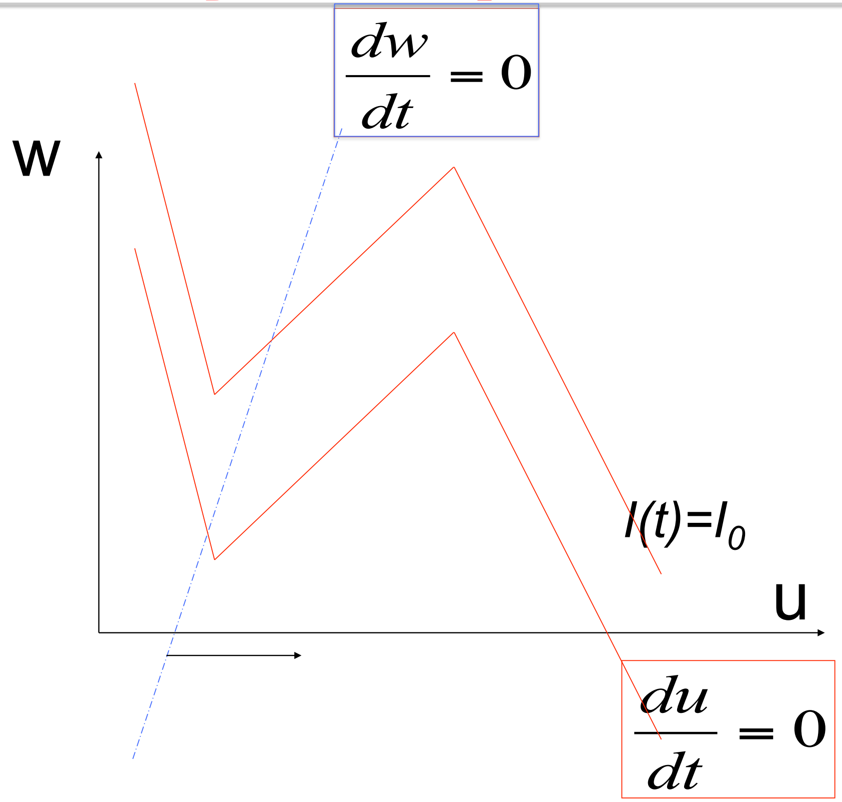
How to determine stability of fixed point?

Neuronal Dynamics – 4.3 Detour. Stability of fixed points

stimulus

$$\tau \frac{du}{dt} = au - w + I_0$$

$$\tau_w \frac{dw}{dt} = cu - w$$

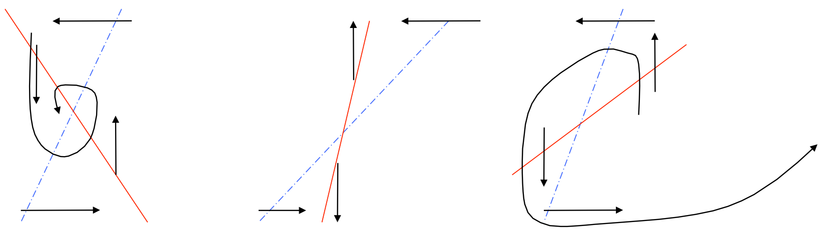


Neuronal Dynamics – 4.3 Detour. Stability of fixed points

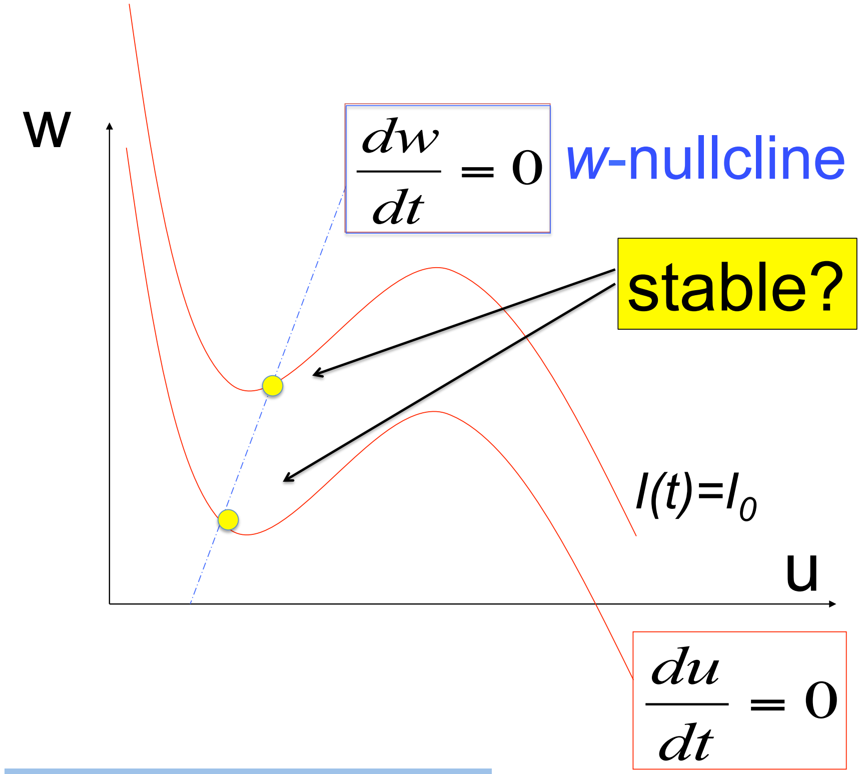
$$\tau \frac{du}{dt} = F(u,w) + RI_0$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

zoom in:



stable saddle unstable



Math derivation now

Neuronal Dynamics – 4.3 Detour. Stability of fixed points

$$\tau \frac{du}{dt} = F(u,w) + RI_0$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

Fixed point at (u_0, w_0)

At fixed point

$$0 = F(u_0, w_0) + RI_0$$

$$0 = G(u_0, w_0)$$

zoom in:

$$x = u - u_0$$

$$y = w - w_0$$

Neuronal Dynamics – 4.3 Detour. Stability of fixed points

$$\tau \frac{du}{dt} = F(u,w) + RI_0$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

Fixed point at (u_0, w_0)

At fixed point

$$0 = F(u_0, w_0) + RI_0$$

$$0 = G(u_0, w_0)$$

zoom in:

$$x = u - u_0$$

$$y = w - w_0$$

$$\tau \frac{dx}{dt} = F_u x + F_w y$$

$$\tau_w \frac{dy}{dt} = G_u x + G_w y$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x} = \begin{pmatrix} F_u & F_w \\ G_u & G_w \end{pmatrix} \boldsymbol{x} \,;$$

Linear matrix equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x} = \begin{pmatrix} F_u & F_w \\ G_u & G_w \end{pmatrix} \boldsymbol{x}\,;$$

Search for solution

$$\boldsymbol{x}(t) = \boldsymbol{e}\,\exp(\lambda t)$$

Two solution with Eigenvalues λ_+, λ_-

$$\lambda_+ + \lambda_- = F_u + G_w$$

$$\lambda_+ \lambda_- = F_u G_w - F_w G_u$$

Neuronal Dynamics – 4.3 Detour. Stability of fixed points

Linear matrix equation

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Search for solution

$$\boldsymbol{x}(t) = \boldsymbol{e}\,\exp(\lambda t)$$

Two solution with Eigenvalues λ_+, λ_-

$$\lambda_+ + \lambda_- = F_u + G_w$$

$$\lambda_+ \lambda_- = F_u G_w - F_w G_u$$

Stability requires:

$$\lambda_+ < 0 \quad and \quad \lambda_- < 0$$

$$F_u + G_w < 0$$

and

$$F_u G_w - F_w G_u > 0$$

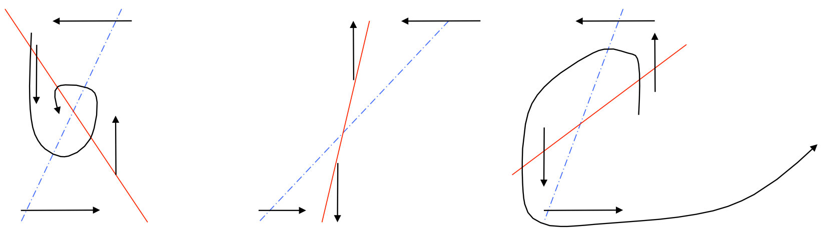
Neuronal Dynamics – 4.3 Detour. Stability of fixed points

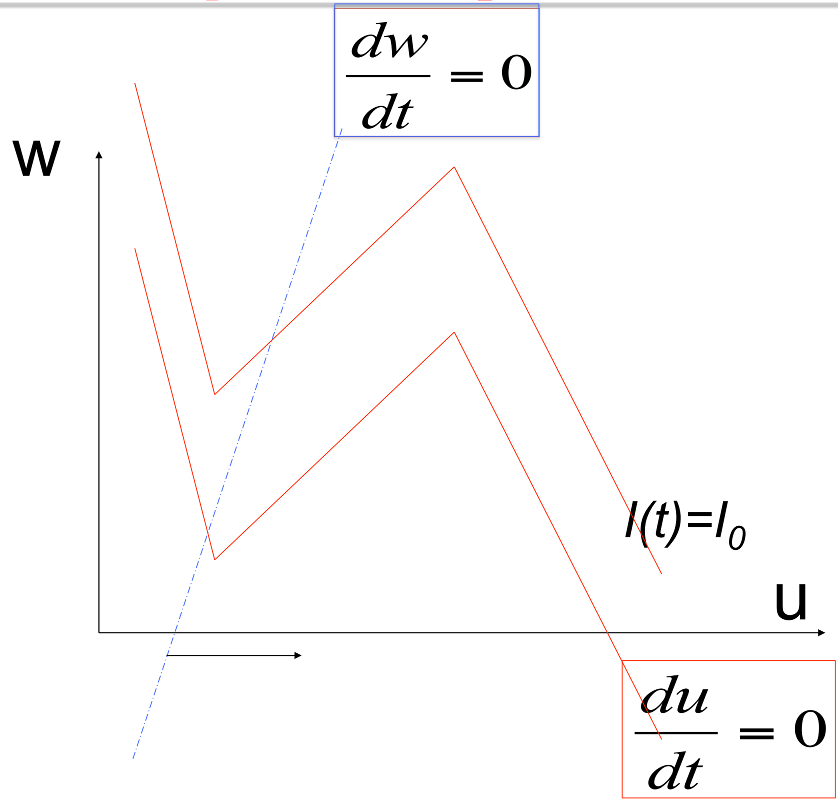
stimulus

$$\tau \frac{du}{dt} = au - w + I_0$$

$$\tau_w \frac{dw}{dt} = cu - w$$

$$\lambda_{+/-} =$$





Neuronal Dynamics – 4.3 Detour. Stability of fixed points

2-dimensional equation

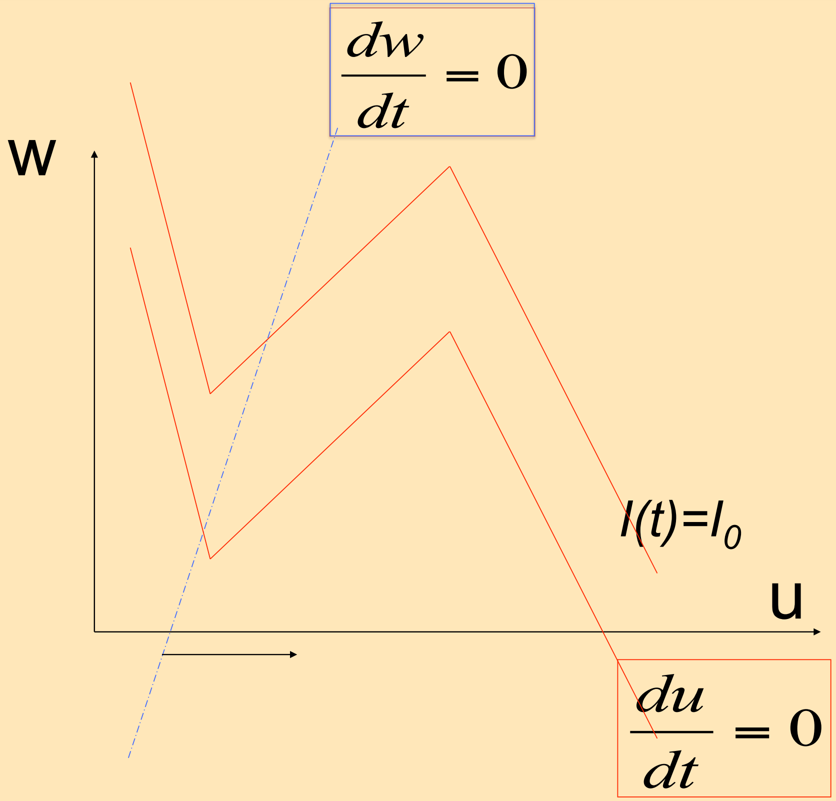
$$\tau \frac{du}{dt} = F(u,w) + RI_0$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

Stability characterized by Eigenvalues of linearized equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x} = \begin{pmatrix} F_u & F_w \\ G_u & G_w \end{pmatrix} \boldsymbol{x}$$

Neuronal Dynamics – Assignment.



Stability analysis of 2-dimensional equations is important for the homework assignment of week 4.