Unit 6: Government policy in competitive markets II – Distribution & incidence

Prof. Antonio Rangel

December 23, 2014

1 Endogenous income and inequality

1.1 Simple model

- There are two markets and two goods:
  - Labor market: consumers sell labor l at wage w to firms
  - Goods market: firms sell good q at price p to consumers
- Consumer side of the model:
  - C consumers
  - no exogenous wealth: $W_i = 0$ for all i
  - $U_i(q, l, m) = q - \frac{l^2}{2\theta_i} + m$
  - l = units of labor provided
  - $\frac{1}{\theta_i}$ = measure of i's disutility of providing labor (i.e., a cost of effort).
  - People with higher θ_i have lower disutility of labor.
  - Consumer's problem:

$$\max_{q,l \geq 0} q - \frac{l^2}{2\theta_i} + lw - pq$$

  - FOCs for good q: $MB = MC$ with $MB = 1$ and $MC = p$.

- It follows that demand for good q is given by

$$x_i^D(p) = \begin{cases} 0 & \text{if } p > 1 \\ anything & \text{if } p = 1 \\ \infty & \text{if } p < 1 \end{cases}$$

- Thus, aggregate demand for the market is given by

$$X_{mkt}^D(p) = \begin{cases} 0 & \text{if } p > 1 \\ anything & \text{if } p = 1 \\ \infty & \text{if } p < 1 \end{cases}$$

- FOCs for labor supplied by consumer i: $MB = MC$ with $MB = w$ and $MC = \frac{l}{\theta_i}$.
- It follows that $l_i^S(w) = \theta_i w$.
- Thus, aggregate labor supply is given by $L_{mkt}^S(w) = w \sum_i \theta_i = wC\bar{\theta}$, where $\bar{\theta}$ denotes average θ.
- Labor income of consumer i: $I_i(w) = \theta_i w^2$

- Firm side of the model:
  - F identical firms
  - Production function: $F(l) = \gamma l$, $\gamma > 0$
  - Firm's problem:

$$\max_{l \geq 0} p\gamma l - wl$$

  - FOCs for firm's problem: $MB = MC$ with $MB = p\gamma$ and $MC = w$.
  - Thus, we get

$$l_j^D(w, p) = \begin{cases} 0 & \text{if } w > p\gamma \\ anything & \text{if } w = p\gamma \\ \infty & \text{if } w < p\gamma \end{cases}$$

- Aggegate labor demand is then given by:

$$L^D_{mkt}(w,p) = \begin{cases} 0 & \text{if } w > p\gamma \\ anything & \text{if } w = p\gamma \\ \infty & \text{if } w < p\gamma \end{cases}$$

- Given this, firms' supply at the individual and market level are given by $q^S_j(w,p) = \gamma l^D_j(w,p)$ and $X^S_{mkt}(w,p) = \gamma L^D_{mkt}(w,p)$

- Competitive market equilibrium:
    - CME given by p^*, w^*, α^* such that :
        1. Consumers optimize over q, l given p^*, w^*
        2. Firms optimize over q, l given p^*, w^*
        3. Both markets clear
    - The CME in this simple model satisfies the following properties:
      $p^* = 1$
      $w^* = \gamma$
      $q^* = C\bar{\theta}\gamma^2$
      $l^* = C\bar{\theta}\gamma$
    - Note that prices and wages are uniquely determined by market parameters, independent of quantity
    - Note that there are many possible CMEs, one for each possible quantity q

- Equilibrium level of inequality
    - In equilibrium, income of person i is given by $I_i = \theta_i ww = \theta_i\gamma^2$
    - This implies that the inequaility of income is given by $Var(I_i) = \gamma^4 Var(\theta_i)$
    - In equilibrium, the utility of person i is given by:

$$\begin{aligned} U_i &= -\frac{l^2}{2\theta_i} + lw + q - pq \\ &= -\frac{\theta_i^2 w^2}{2\theta_i} + \theta_i w^2 \\ &= \frac{\theta_i\gamma^2}{2} \end{aligned}$$

– It follows that the inequality of utility is given by $Var(U_i) = \frac{\gamma^4}{4}Var(\theta_i)$

- Summary: a simple model with individual differences in the disutility of labor generates inequality of income and utility in equilibrium

1.2 Labor income taxes and inequality

- Impact of taxes on market equilibrium

  – Labor income tax $\tau > 0$: for every dollar each consumer earns, she must pay τ to government

  – Revenue returned to consumers using IDENTICAL lump sum transfer $T = \frac{\text{revenue raised}}{C}$

  – Consumer's problem now given by:

$$\max_{q,l \geq 0} q - \frac{l^2}{2\theta_i} + l(1-\tau)w - pq + T$$

  – Consumer assumes T fixed when maximizing, due to large number of individuals in market

  – No change in q-market $\implies p^* = 1$

  – No change in firms' problem $\implies w^* = \gamma$

  – Labor supply now given by $MB = MC$, with $MB = (1-\tau)w$ and $MC = \frac{l}{\theta_i}$, which implies that

$$l_i^S(w) = (1-\tau)w\theta_i$$

  and

$$L_{mkt}^S(w) = (1-\tau)wC\bar{\theta}$$

- Impact of taxes on total tax revenue:

  – $TotalRev(\tau) = \tau w L_{mkt}^S(w^*) = \tau(1-\tau)\gamma^2 C\bar{\theta}$

  – The relationship of total Revenue vs. the tax rate is often called a Laffer curve

- Key property of the Laffer curve: tax revenue increases with tax rate when τ is small, but decreases with the tax rate when τ is sufficiently large

- Impact of taxes on redistribution:
  - Net-Tax = transfers received - taxes paid
  - For individual i, we have that

$$\begin{aligned} NetTax(\theta_i) &= \tau(1-\tau)\gamma^2\bar{\theta} - \tau(1-\tau)\gamma^2\theta_i \\ &= \tau(1-\tau)\gamma^2\left[\bar{\theta} - \theta_i\right] \end{aligned}$$

  - $NetTax(\theta_i) > 0$ if and only if $\theta_i < \bar{\theta}$
  - This implies that individuals with an above averege disutility of labor receive a net transfer from the government, and those with below average disutility of labor pay a net tax.

- Impact of taxes on income inequality :
  - $I_i = (1-\tau)^2\gamma^2\theta_i + \tau(1-\tau)\gamma^2\bar{\theta}$
  - Tax reduces income inequality: intercept increasing, slope decreasing in τ
  - $Var(I_i) = (1-\tau)^4\gamma^4 Var(\theta_i)$

- Impact of taxes on utility inequality:
  - Post-tax utility of person i is given by

$$\begin{aligned} U_i(\tau) &= -\frac{(1-\tau)^2\gamma^2\theta_i^2}{2\theta_i} + (1-\tau)^2\gamma^2\theta_i + \tau(1-\tau)\gamma^2\bar{\theta} \\ &= \frac{(1-\tau)^2\gamma^2\theta_i}{2} + \tau(1-\tau)\gamma^2\bar{\theta} \\ &= \frac{1}{2}\gamma^2(1-\tau)\left[\theta_i + \tau(2\bar{\theta} - \theta_i)\right] \end{aligned}$$

  - $Var(U_i) = \frac{(1-\tau)^4\gamma^4}{4} Var(\theta_i)$

- Lesssons:
  - An income tax can reduce, but not eliminate, inequality
  - There is an efficiency cost of income taxes, since people work less as the tax rate increases

1.3 Optimal labor income tax

- Now let's compute the optimal labor income tax in our model
- Suppose consumers have preferences that depend on the distribution of income, as follows:

$$V_i(q, l, m, \{I_1, \ldots, I_C\}) = \underbrace{\left[q - \frac{l^2}{2\theta_i} + m\right]}_{U_i} - \underbrace{\sigma\sqrt{Var(I)}}_{\text{Inequality pref}}$$

- Optimal tax problem for the government:

$$\max_{\tau \geq 0} \sum_i V_i(\tau)$$

- Notation:
  - $U_i(\tau), V_i(\tau)$: utility as a function of tax rate, as derived above
  - $n(\theta)$: number of consumers of type θ
  - $I_\theta(\tau)$: total income of consumer of type θ as a function of tax rate
- Simplifying objective function we get

$$\begin{aligned}\sum_i V_i(\tau) &= \sum_i U_i(\theta) - C\sigma\sqrt{Var(I_\theta(\tau))} \\ &= \sum_\theta n(\theta)\left[\frac{w^2(1-\tau)^2\theta}{2} + \tau(1-\tau)w^2\bar{\theta}\right] - C\sigma\sqrt{\frac{\sum n(\theta)\left(I_\theta(\tau) - \overline{I_\theta(\tau)}\right)^2}{C}}\end{aligned}$$

- Observe that

$$I_\theta(\tau) - \overline{I_\theta(\tau)} = (1-\tau)^2 w^2(\theta_i - \bar{\theta})$$

- This implies that

$$\begin{aligned}\sum n(\theta)\left(I_\theta(\tau) - \overline{I_\theta(\tau)}\right)^2 &= (1-\tau)^4 w^4 \sum n(\theta)(\theta_i - \bar{\theta})^2 \\ &= (1-\tau)^4 w^4 Var(\theta) C\end{aligned}$$

- So the objective function $\sum_i V_i(\tau)$ can be written as

$$
\begin{aligned}
&= \frac{w^2(1-\tau)^2}{2}\underbrace{\sum \theta n(\theta)}_{=C\bar{\theta}} + \tau(1-\tau)w^2\bar{\theta}\underbrace{\sum n(\theta)}_{=C} - C\sigma(1-\tau)^2 w^2 SD(\theta) \\
&\propto w^2 C\bar{\theta}\left[(1-\tau)^2 + 2\tau(1-\tau) - 2(1-\tau)^2\sigma\frac{SD(\theta)}{\bar{\theta}}\right] \\
&\propto (1-\tau^2) - 2(1-\tau)^2\sigma\frac{SD(\theta)}{\bar{\theta}}
\end{aligned}
$$

- As a result, the optimal tax problem can be written as

$$\max_{\tau\geq 0}(1-\tau^2) - 2(1-\tau)^2\sigma\frac{SD(\theta)}{\bar{\theta}}$$

- From the Laffer curve material, we know that this problem has a unique maximum. So the following FOCs are necessary and sufficient:

$$-2\tau + 4(1-\tau)\sigma\frac{SD(\theta_i)}{\bar{\theta}} = 0$$

- This implies a precise formula for the optimal tax

$$\tau^{opt} = \frac{2\sigma\frac{SD(\theta_i)}{\bar{\theta}}}{1 + 2\sigma\frac{SD(\theta_i)}{\bar{\theta}}}.$$

- Intuition check:
  - $\tau^{opt} = 0$ when individuals don't care about inequality since $\sigma = 0$
  - $\tau^{opt} = 0$ when there is no inequality since $SD(\theta) = 0$
  - The optimal tax goes to 1 as the distate for inequality increases (i.e., as $\sigma \to \infty$)

- Remark 1: Optimal tax problem induces a fundamental tradeoff: reduce inequality vs. avoid inefficiency

- Remark 2: Is the result robusts to alternative model specifications? Basic logic of the problem is robust, although the precise details of the formula depends on the details of the model.

- Remark 3: Solution depends on consumers' objective function. We used an objective function in which consumers care about overall inequality, not about others' utility.
- Remark 4: Key empirical parameters affecting the size of the optimal tax:
  - measure of inequality $\frac{SD(\theta)}{\bar{\theta}}$
  - strength of social preferences σ
  - general equilibrium effects of taxation

1.4 Second welfare theorem

- Second Welfare Theorem (SWT):
  - Let α be a Pareto optimal allocation
  - Then there is a set of lump-sum $m-$good transfers, with $\sum T_i = 0$, such that α is a CME given transfers $\{T_i\}$
- Intuition: The market for the q-good is not affected by lump-sum transfers.
- Naive interpretation of the SWT:
  - It implies no need to use distortionary taxes to redistribute.
  - It implies usage of lump-sum transfers to reach desired P.O. allocation, since they generate no DWL!
- Problem – Lump-sum tax policy in SWT involves unreasonable informational demands:
  - must choose T_i for each consumer
  - therefore must know fundamental parameters (preferences, effort costs) of each individual
  - very unrealistic!

2 Price controls

2.1 Simple price controls

- Taxonomy of price control policies
  - Price ceiling: $p \leq p^{max}$
  - Price floor: $p \geq p^{min}$
  - Simple: just price restriction
  - Complex: price restriction plus action necessary to clear market
- When does a simple price ceiling affect equilibrium outcomes?
  - Policy is not binding if $p^{max} \geq p^*$. In this case the market generates the same outcome
  - Policy binds if $p^{max} < p^*$. In this case p^{max} becomes the equilibriu price. But at that price there is excess demand, so a rationing rule is needed (specifying who gets the units that are produced)
  - Efficient rationing rule: units are allocated to highest-value consumers
- Effect of binding price ceiling on social surplus under an efficient rationing rule:

	free mkt	p^{max}	change
CS	A + B + E	A + B + C	C - E
PS	C + D + F	D	-(C+F)
SS	A + ... + F	A + B + C + D	-(E+F)

- Note: E+F represents the DWL introduced by the price-ceiling policy
- Remarks:
  - When policy binds, it creates inefficiency
  - Policy can have redistributive effects. For example, in some cases there is a transfer of surplus from firm owners to consumers who are not firm owners.

2.2 Complex price controls

- Complex price floor
  - $p \geq p_{min}$
  - Government buys excess supply at equilibrium price p^*
  - Revenue for government purchases financed using an equal lump-sum tax in all consumers
  - Units bought by government are destroyed

- Effects of binding complex price floor on equilibrium outcomes
  - Important quantities: $p^* = p_{min}$, $x^*_{consumed}$, $x^*_{produced}$
  - $x^*_{consumed} < x^*_{produced}$
  - Government buys excess production and destroys it

- Effect of binding complex price floor on social surplus:

	free mkt	p^{min}	change
CS	A+B+E	A-(E+F+G+H+I)	-(B+2E+F+G+H+I)
PS	C+D+F	B+C+D+E+F+I	B+E+I
SS	A+B+C+D+E+F	A+B+C+D-(H+G)	-(E+F+H+G)

- NOTE: E+F+H+G represents the DWL of the policy
- Remarks:
  - DWL bigger than in simple price control
  - Policy also entails a trasfer of surplus fron consumers to owners of the firms
  - This policy is particularly inefficient!

3 Economic vs. legal incidence

- Does it matter who pays the tax?
  - Suppose that government needs to raise a tax $\tau > 0$ per unit of good q sold:
  - Tax can be assigned to consumers, producers, or both
  - Class of policies:
    * $0 \le a \le 1$: a is fraction of tax paid by consumers
    * tax on consumers per-unit purchased: $\tau^C = a\tau$
    * tax on producers per-unit sold: $\tau^F = (1-a)\tau$
- Incidence:
  - Legal: who sends a check to the government
  - Economic: who bears the cost of the tax
- RESULT: Equilibrium allocation is independent of a
  - $p =$ mkt price
  - $p + a\tau$: net price paid by consumers
  - $p - (1-a)\tau$: net price received by firms
  - Consumers treat tax as price increase: $X^D(p|a) = X^D_{no-tax}(p+a\tau)$
  - Likewise for producers: $X^S(p|a) = X^S_{no-tax}(p-(1-a)\tau)$
  - Market equilibrium $p^*(a)$ solves

$$X^D_{no-tax}(p^*(a) + a\tau) = X^S_{no-tax}(p^*(a) - (1-a)\tau)$$

  - Let $p^*_\tau =$ equilibrium price when $a = 0$ (all tax paid by firms)
  - Easy to check that $p^*(a) = p^*_\tau - a\tau$ clears the market for all a
  - But then the net price paid by consumers and received by firms is independent of a
  - This immplies that the equilibrium allocation is also independent of a!
- See graphical intuition provided in video lecture

4 Final remarks

- Key ideas from this unit:

  1. Optimal tax problem involves a tradeoff between redistribution and inefficiency
  2. Price controls lead to sizeable deadweight losses, but can improve consumer or producer surplus, through the redistribution of social surplus
  3. Legal incidence $\neq$ economic incidence