Unit 5: Government policy in competitive markets I – Efficiency

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1 Pareto optimal allocations

1.1 Preliminaries

- Big picture
  - Consumers: $1, \ldots, C$, each w/ U_i, W_i
  - Firms: $1, \ldots, F$, each w/ $C_j(\cdot)$
  - Consumers and firms interact through an institution (e.g., the market) to produce a feasible allocation α
  - Big question: Is the resulting allocation α desirable?
- Three notions of desirability:
  1. Efficiency: measures lack of waste in production/consumption
  2. Distributive justice: measures fairness of distribution of resources, e.g. maximin
  3. Procedural justice: measures fairness of process used to reach allocation, e.g. equal treatment
- In this course, we look at 1 and 2. Procedural justice is studied in more advanced courses.

1.2 Pareto optimality

- Pareto improvement test: A feasible allocation α^1 Pareto improves over feasible allocation α^0 if:

  1. $U_i(\alpha_i^1) > U_i(\alpha_i^0)$ for at least one i
  2. $U_i(\alpha_i^1) \geq U_i(\alpha_i^0)$ for all i

  I.e., need to make somebody better off without making anybody worse off

- A feasible allocation α is Pareto optimal (PO) if there doesn't exist another feasible allocation $\hat{\alpha}$ that Pareto improves over α.

- Example 1:

  - 2 consumers with preferences $U(q, m) = m$
  - W_i of good m
  - No production
  - Feasible allocation: $m_1 + m_2 = W_1 + W_2$.
  - All feasible allocations Pareto Optimal
  - This illustrates orthogonality between PO and equality

- Example 2 :

  - $C = F = 1$
  - Look at cost function with $SFC > 0$, but $FC = 0$
  - See video for graphical characterization of the set of feasible and PO allocations.

1.3 Properties of Pareto optimal allocations

- RESULT: Let α be a feasible allocation. Then the following statements are equivalent (under the mainatined assumption that preferences are quasi-linear and m is unbounded below):

  - α is Pareto optimal

- α solves $\max\limits_{\gamma \text{ feasible}} \sum\limits_i U_i(\gamma_i)$

- Proof outline:
  - If not PO then there exists a feasible Pareto improving allocation, which implies that it also increases $\sum U_i$
  - If not max $\sum U_i$, then there exists a feasible allocation β which yields higher $\sum U_i$. Then possible to construct another feasible allocation γ, by redistributing m from the allocation β, that makes everyone strictly better off. This implies that γ is a Pareto improvement over the initial allocation.

- RESULT: Let α be a feasible allocation. Then the following are equivalent (under the mainatined assumption that preferences are quasi-linear and m unbounded below):
  - α is Pareto optimal
  - α solves $\max\limits_{\gamma \text{ feasible}} SS(\gamma)$

- Proof:
  - From above, P.O $\sim \max_\gamma \sum U_i(\gamma_i)$
  - From Unit 4, $\max_\gamma \sum U_i(\gamma_i) \sim \max_\gamma SS(\gamma) + constant$

- REMARK: Properties of a PO allocation don't depend on distribution of m

- REMARK: To find set of PO allocations, solve

$$\max_{\substack{q_1^c, \ldots, q_C^c \\ q_1^f, \ldots, q_F^f}} \sum_i B_i(q_i^c) - \sum_j C_j(q_j^f)$$

$$\text{s.t.} \sum_i q_i^c = \sum_j q_j^f$$

- Necessary conditions for PO:

1. Efficient allocation of production to firms:
   For any j, h, k with $q_j^f > 0, q_h^f > 0$, and $q_k^f = 0$,

$$C_j'(q_j^f) = C_h'(q_h^f) < C_k'(q_k^f)$$

2. Efficient allocation of consumption:
   For any i, j, k with $q_i^c > 0, q_j^c > 0$, and $q_k^c = 0$,

$$B_i'(q_i^c) = B_j'(q_j^c) > B_k'(q_k^c)$$

3. Overall production efficiency:
   For any i, j with $q_i^c > 0$ and $q_j^f > 0$,

$$B_i'(q_i^c) = C_j'(q_j^f)$$

- Intuition for Condition 1:
  - Suppose $C_j'(q_j^f) > C_h'(q_h^f)$, for firms i, j with $q_j^f, q_h^f > 0$,
  - Then can decrease q_j^f by dq and increase q_h^f by dq
  - This leaves total production unchanged, while decreasing total costs of production

- Intuition for Condition 2:
  - Suppose $B_i'(q_i^c) < B_j'(q_j^c)$, for consumers with $q_i^c, q_j^c > 0$
  - Then can transfer dq from i to j and $dm = dqB_i'(q_i^c)$ from j to i
  - i indifferent between old and new allocations
  - j strictly better off since $dU_j = dq(B_j'(q_j^c) - B'(q_i^c)) > 0$

- Intuition for Condition 3:
  - Suppose $B_i'(q_i^c) > C_j'(q_j^f)$ at an interior point
  - Firm j can produce dq more and give it to consumer i in exchange for $dm = dqC_j'(q_j^f)$
  - This is feasible, because the dm transfer covers exactly the extra costs of the firm

- But consumer's utility increases since $dU_i = dqB_i'(q_i^c) - dqC_j'(q_j^f) > 0$

- Sufficient conditions for PO:
  - Difficult problem to derive general sufficient conditions, especially in presence of FCs or SFCs
  - If $MB > 0, MB \downarrow$ and $C(\cdot)$ is DRS, then the three necessary conditions for PO given above are also sufficient

1.4 Example

- Example of interior PO allocation
  - $C = F = 1$
  - Cost function: DRS, $FC = SFC = 0$
  - Feasible set is graph of points $(q, W - c(q))$ in qm-plane, w/ slope $-c'(q)$
  - Indifference curves satisfy $B'(q)dq + dm = 0$, so have slope $-B'(q)$
  - PO allocations satisfy $B'(q^c) = c'(q^f)$ (Condition 3)

- Example of corner PO allocation
  - As before:
    * $C = F = 1$
    * Cost function: DRS, $FC = SFC = 0$
    * Feasible set is graph of points $(q, W - c(q))$ in qm-plane, w/ slope $-c'(q)$
    * Indifference curves satisfy $B'(q)dq + dm = 0$, so have slope $-B'(q)$
  - However, indifference curves nearly flat and cross m-axis with $-c'(0) < -B'(0)$
  - $c'(0) > B'(0)$, so no improvement possible from $(0, W)$ and this is the unique PO allocation

1.5 Example

- $C = 10$, $U_i(q, m) = \alpha_i \ln(q) + m$, $\alpha_i > 0$, W_i
- $F = 10$, $C_j(q) = \beta_j q^2$, $\beta_j > 0$
- Condition 1: $MC_j = 2\beta_j q_j^f \implies$
  1. Interior solutions
  2. For all pairs of firms k, j:

     $$2\beta_j q_j^f = 2\beta_k q_k^f \implies \frac{q_j^f}{q_k^f} = \frac{\beta_k}{\beta_j}$$

     i.e. firms with lower marginal costs produce more
- Condition 2: $MB_i = \frac{\alpha_i}{q_i^c} \implies$
  1. interior solutions
  2. For all pairs of consumer i, j:

     $$\frac{\alpha_i}{q_i^c} = \frac{\alpha_j}{q_j^c} \implies \frac{q_i^c}{q_j^c} = \frac{\alpha_i}{\alpha_j}$$

     i.e. consumers with higher marginal benefit consume more
- Condition 3: $MB_i = MC_j$ for all $i, j \implies$

  $$\frac{\alpha_i}{q_i^c} = 2\beta_j q_j^f$$

  for any consumer-firm pair
- Feasibility constraint:

  $$\sum_i q_i^c = \sum_j q_j^f. \tag{1}$$

- By Condition 1,

  $$\sum_i q_i^c = \sum_i \frac{\alpha_i}{\alpha_1} q_1^c$$

  and

  $$\sum_j q_j^f = \sum_j \frac{\beta_1}{\beta_j} q_1^f$$

- So (1) is equivalent to

$$\sum_i \frac{\alpha_i}{\alpha_1} q_1^c = \sum_j \frac{\beta_1}{\beta_j} q_1^f. \tag{2}$$

- Set $A = \sum_i \frac{\alpha_i}{\alpha_1}$ and $B = \sum_j \frac{\beta_1}{\beta_j}$ and write (2) as

$$A q_1^c = B q_1^f. \tag{3}$$

- From Condition 3, we have

$$\frac{\alpha_1}{q_1^c} = 2\beta_1 q_1^f. \tag{4}$$

- Solving (3) and (4) for q_1^f and q_1^c yields

$$q_1^f = \sqrt{\frac{\alpha_1}{2B\beta_1}}$$

$$q_1^c = \sqrt{\frac{B\alpha_1}{2A\beta_1}}$$

- These are the quantities firm 1 must produce and consumer 1 must consume in a PO allocation. From here we can identify the full allocation by using the expressions above.

- NOTE: The allocation of q, but not of m, is uniquely determined in PO allocation. Since all consumers have same MB for m, we can shift m around without affecting the optimality of the allocation.

2 First Welfare Theorem

2.1 Result

- First Welfare Theorem (FWT): Any competitive market equilibrium allocation is Pareto optimal

- Proof:

- By contradiction
- Let α^*, p^* be a cME
- $p^* > 0$, since otherwise some consumer would consume an infinite amount of q
- Suppose that α^* is not PO
- Then there exists another feasible β which is Pareto improving.
- This implies that:
  1. for at least one i, $U_i(\beta_i) > U_i(\alpha_i^*)$
     $\implies p^* q_i^c(\beta) + m_i^c(\beta) > p^* q_i^c(\alpha^*) + m_i^c(\alpha^*)$,
     i.e., what i consumes at β must cost more that what she consumed at α^*, since otherwise the consumer would not have been maximizing her utility at α^*.
  2. For every consumer j, $U_j(\beta_j) \geq U_j(\alpha_j^*)$
     $\implies p^* q_j^c(\beta) + m_j^c(\beta) \geq p^* q_j^c(\alpha^*) + m_j^c(\alpha^*)$, by a parallel argument.
- Together, this implies:

$$\sum_i p^* q_i^c(\beta) + \sum_i m_i^c(\beta) > \sum_i p^* q_i^c(\alpha^*) + \sum_i m_i^c(\alpha^*) \tag{5}$$

- By feasibility,

$$\sum_i p^* q_i^c(\beta) + \sum_i m_i^c(\beta) = \sum_i p^* q_i^c(\beta) + \sum_i W_i - \sum_j C_j(q_j^f(\beta))$$

  and

$$\sum_i p^* q_i^c(\alpha^*) + \sum_i m_i^c(\alpha^*) = \sum_i p^* q_i^c(\alpha^*) + \sum_i W_i - \sum_j C_j(q_j^f(\alpha^*))$$

- It follows that (5) is equivalent to

$$\sum_i p^* q_i^c(\beta) - \sum_j C_j(q_j^f(\beta)) > \sum_i p^* q_i^c(\alpha^*) - \sum_j C_j(q_j^f(\alpha^*)),$$

  after cancelling the W_i terms from both sides.

- Since the sum of consumers' expenditures must equal the sum of firms' revenues, this is equivalent to

$$\sum_j \Pi_j(\beta) > \sum_j \Pi_j(\alpha *),$$

  where Π = profits.

- But this means at least one firm earns higher profits at β than they did at α^*, which contradicts the assumption that firms maximize profits at the cME.
- Therefore α^* is not a CME, which is a contracition.

• Intuition 1: "The invisible hand of the market"

  - Market forces: market settles at p^*, α^* with:
    1. α^* is feasible (market clearing)
    2. $MB_i = p^*$ from utility maximization by consumers
    3. $p^* = MC_j$ from profit maximization by firms
  - But this induces the necessary conditions for PO, even though consumers only care about maximizing their own utility, and firms only care about maximizing their own profits:
    1. since $MC_k = p^* = MC_j$
    2. since $MB_i = p^* = MB_j$
    3. since $MB_i = p^* = MC_j$
  - Bottom line: If firms and consumers all try to do their best, ignoring each other's needs, market forces will lead them to settle on an allocation that satisfies PO!!!!!!!!!!!

• Intuition 2:

  - RESULT: SS maximized over feasible allocations at a cME allocation α^*
  - Proof:
    * FWT $\implies$ α^* is P.O.

        * PO$\sim \max\limits_{\gamma feasible} SS(\gamma)$ (under the maintained assumptions of quasi-linear preferenes and m unbounded below)

  - Critical graph: Please see video lecture.
  - The equilibrium market quantity X^* is the optimal level of production of good x
  - The equilibrium market price p^* equals the marginal social benefit *and* the marginal social cost of producing and consuming another unit

2.2 Discussion

- Remark 1: FWT underlies economists' widespread belief in free-markets.
- Remark 2: FWT shows that competitive markets lead to PO allocations at very low informational demands.
  Required information:
  - Consumers: know only their own $U(\cdot)$ and p^*
  - Producers: know only their own $C(\cdot)$ and p^*
- Compare to required information for a dictator or central planner:
  - Need to know all $U(\cdot)$s
  - Need to know all $C(\cdot)$s
  - Then must solve a computationally difficult optimization problem
- Most economists believe that the difficulty of accurately gathering the required information and then solving the necessary optimization problem is the main reason centrally planned economies like the Soviet Union have typically failed to produce the same levels of economic growth as free market economies.
- Key assumptions behind the FWT, and consequences when they fail:

Assumptions	Consequences
Optimal decision making by consumers and firms	FWT fails due to DM mistakes (e.g. marketing, myopia)
Every actor is a price taker	Imperfect competition, FWT fails (monopoly, oligopoly, brands)
No externalities	Public goods & externalities, FWT fails (environment, R&D)
Perfect information	Asymmetric information, FWT fails (insurance, used cars, contracts)

3 Taxes and efficiency

3.1 Deadweight loss

- Let SS^{opt}, α^{opt} denote the solution to the social surplus max problem over all feasible allocations
- The Deadweight Loss (DWL) is given by:

$$\begin{aligned} DWL(\alpha) &= SS^{opt} - SS(\alpha) \\ &= \text{measure of inefficiency at allocation } \alpha \text{ [in \$s]} \\ &= \sum_i U_i(\alpha^{opt}) - \sum_i U_i(\alpha) \\ &= \left[\sum_i B_i(q_i^{c,opt}) - \sum_j C_j(q_j^{f,opt})\right] - \left[\sum_i B_i(q_i^{c,\alpha}) - \sum_j C_j(q_j^{f,\alpha})\right] \end{aligned}$$

- Graphical representation:
  - Assume that any quantity is produced is allocated efficiently among producers and consumers
  - This leads to an important graphical represenation of the DWL. (See video for details)
- REMARK: DWL increases non-linearly (and often as the square) with deviations from q^{opt}

3.2 Lump-sum taxes

- Basic taxonomy of taxes
  - Lump sum: Specifies fixed amount to be paid by each consumer/firm independent of their actions
    $T > 0$: taxes
    $T < 0$: transfers
  - Non-lump sum: Tax owed depends on consumer/firm actions
    Example: p/unit sales tax paid by consumer
- Note: to isolate the efficiency effects of tax policies, we focus on revenue neutral tax policies in which all revenue raised is returned to consumers using lump-sum taxes
- RESULT: Lump-sum taxes do not introduce inefficiencies
  - Suppose $T = \left(T_1^c, \ldots, T_C^c, T_1^f, \ldots, T_F^f\right)$ with $\sum_i T_i^c + \sum_j T_j^f = 0$
  - Claim: $DWL(\alpha_T) = 0$
  - Why?
  - Consumers: $\max_{q \geq 0} B_i(q) + W_i - T_i^c - pq \implies X_T^D = X_{noT}^D$
  - Firms: $\max_{q \geq 0} pq - C_j(q) - T_j^f \implies X_T^S = X_{noT}^S$
  - I.e., lump-sum taxes do not affect optimization problem for consumer or firm, so cME unchanged
  - Therefore $q_T^* = q_{noT}^* = q^{opt}$, so $DWL(\alpha_T) = 0$.
- Unfortunately, lump-sum taxes have serious limitations, as we'll see later in the course

3.3 Per-unit taxes

- Look at impact of per-unit tax on demand and supply
  - τ p/unit sales tax on consumers [$]
  - revenue returned uisng a lump-sum transfer: $T = \frac{q_\tau^* \tau}{C}$
  - $X_\tau^S = X_{no\tau}^S$

- Consumer: $\max_{q \geq 0} B(q) + W + T - q(p+\tau)$
  $\implies X_{\tau}^{D}(p) = X_{no\tau}^{D}(p+\tau)$
- This assumes that C is very large, so consumers' take the size of the lump-sum transfer as fixed

- Equilibrium effects and DWL
  - Aggregate supply curve stays the same
  - Aggreagate demand curve shifts down by τ
  - Therefore: $q_{\tau}^{*} < q_{no\tau}^{*}$ and $p_{\tau}^{*} < p_{no\tau}^{*}$

- Comparative statics
  - At equilibrium: $X_{no\tau}^{D}\left(p^{*}(\tau)+\tau\right) = X_{no\tau}^{S}\left(p^{*}(\tau)\right)$

$$\implies \frac{dX_{no\tau}^{D}}{dp}\left(\frac{dp^{*}}{d\tau}+1\right) = \frac{dX_{no\tau}^{S}}{dp}\frac{dp^{*}}{d\tau}$$

$$\implies \frac{dp^{*}}{d\tau} = \frac{\frac{dX_{no\tau}^{D}}{dp}}{\frac{dX_{no\tau}^{S}}{dp} - \frac{dX_{no\tau}^{D}}{dp}} < 0$$

  - This formula shows how responses to taxes depend on the relative sensitivity of aggregate demand and aggregate supply to price

4 Final remarks

- Key concepts:
  1. Feasible allocation α is PO if there isn't another feasible β with
     $U_i(\beta_i) > U_i(\alpha_i)$ for some i
     $U_i(\beta_i) \geq U_i(\alpha_i)$ for all i
  2. FWT: allocations generated by competitive markets are PO
  3. Lump-sum taxes redistribute without introducing inefficiencies
     Non lump-sum taxes are distortionary (i.e. they have DWL > 0)