Week 4 – part 2: Phase Plane Analysis

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Neuronal Dynamics:
Computational Neuroscience of Single Neurons

Week 4 – Reducing detail:
Two-dimensional neuron models

Wulfram Gerstner

EPFL, Lausanne, Switzerland

√ 4.1 **From Hodgkin-Huxley to 2D**
- Overview: From 4 to 2 dimensions
- MathDetour 1: Separation of time scales
- MathDetour 2: Exploiting similarities

4.2 **Phase Plane Analysis**
- Role of nullcline

4.3 **Analysis of a 2D Neuron Model**
- MathDetour 3: Stability of fixed points

4.4 **Type I and II Neuron Models**
- where is the firing threshold?
- separation of time scales

4.5. **Nonlinear Integrate-and-fire**
- from two to one dimension

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Neuronal Dynamics – 4.2. Phase Plane Analysis

2-dimensional equation

stimulus

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

Enables graphical analysis!

-Discussion of threshold

-Type I and II

Neuronal Dynamics – 4.1. Reduction of Hodgkin-Huxley model

$$C\frac{du}{dt} = \underbrace{-g_{Na}\, m_0(u)^3 (1-w)(u-E_{Na})}_{I_{Na}} \underbrace{- g_K (\frac{w}{a})^4 (u-E_K)}_{I_K} \underbrace{- g_l (u-E_l)}_{I_{leak}} + I(t)$$

$$\frac{dw}{dt} = -\frac{w - w_0(u)}{\tau_w(u)}$$

stimulus

$$\tau\frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w\frac{dw}{dt} = G(u,w)$$

Neuronal Dynamics – 4.2. Nullclines of reduced HH model

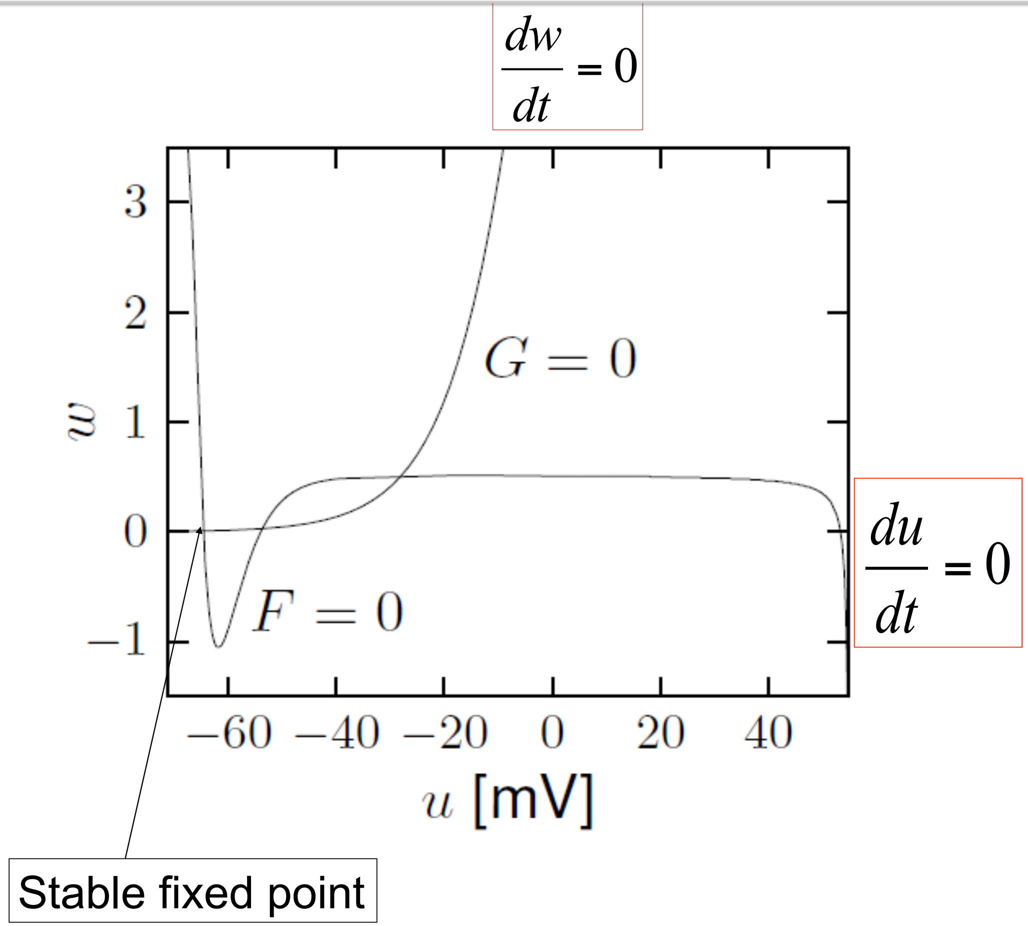
stimulus

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

u-nullcline

w-nullcline

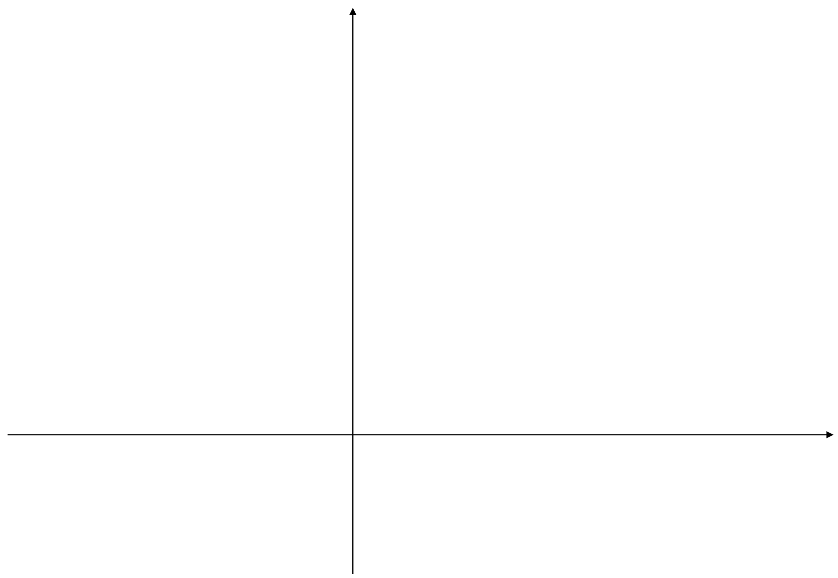


Neuronal Dynamics – 4.2. FitzHugh-Nagumo Model

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

$$= u - \frac{1}{3}u^3 + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w) = b_0 + b_1 u - w$$



u-nullcline

w-nullcline

Neuronal Dynamics – 4.2. flow arrows

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

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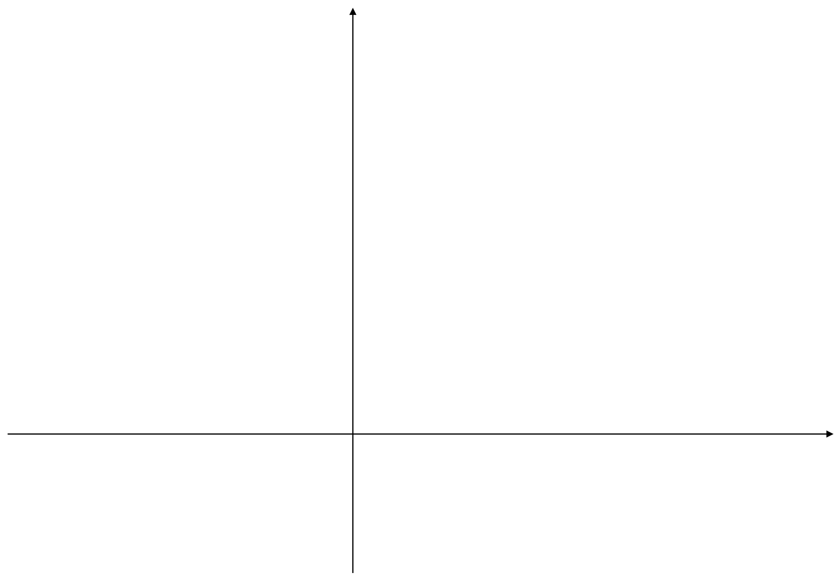


u-nullcline

w-nullcline

Neuronal Dynamics – 4.2. flow arrows

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

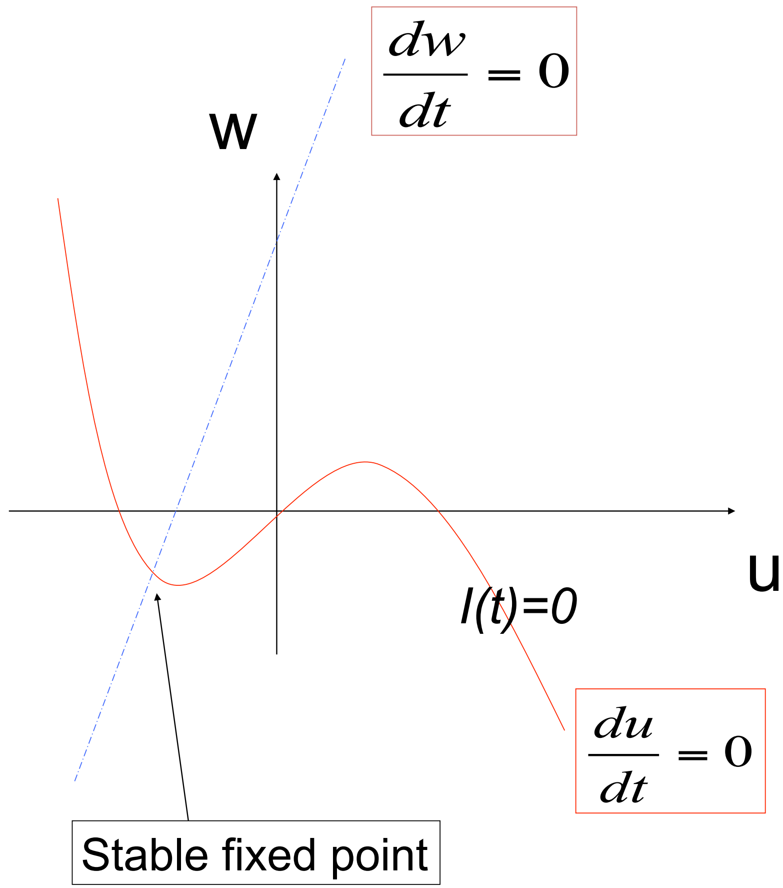
Stimulus I=0

$$\tau_w \frac{dw}{dt} = G(u,w)$$

Consider change in small time step

Flow on nullcline

Flow in reginons between nullclines



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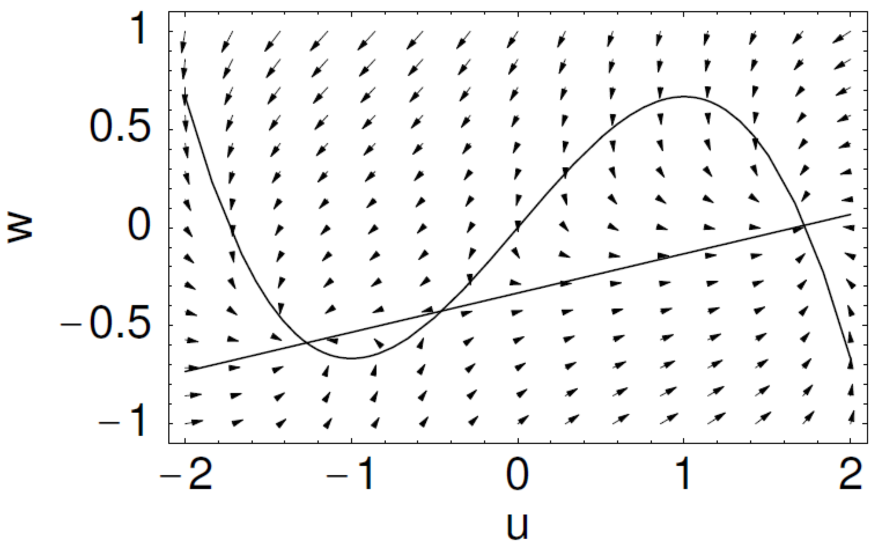

Neuronal Dynamics – 4.2. Phase Plane Analysis

2-dimensional equation

stimulus

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

Enables graphical analysis!

Important role of

- nullclines
- flow arrows

Neuronal Dynamics – Quiz 4.4.

A. u-Nullclines

[ ] On the u-nullcline, arrows are always vertical
[ ] On the u-nullcline, arrows point always vertically upward
[ ] On the u-nullcline, arrows are always horizontal
[ ] On the u-nullcline, arrows point always to the left
[ ] On the u-nullcline, arrows point always to the right

B. w-Nullclines

[ ] On the w-nullcline, arrows are always vertical
[ ] On the w-nullcline, arrows point always vertically upward
[ ] On the w-nullcline, arrows are always horizontal
[ ] On the w-nullcline, arrows point always to the left
[ ] On the w-nullcline, arrows point always to the right
[ ] On the w-nullcline, arrows can point in an arbitrary direction