Week 4 – part 1 : Reduction of the Hodgkin-Huxley Model

Neuronal Dynamics:
Computational Neuroscience of Single Neurons

Week 4 – Reducing detail: Two-dimensional neuron models

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Week 4 – part 1 : Reduction of the Hodgkin-Huxley Model

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

4.1 **From Hodgkin-Huxley to 2D**
- Overview: From 4 to 2 dimensions
- MathDetour 1: Separation of time scales
- MathDetour 2: Exploiting similarities

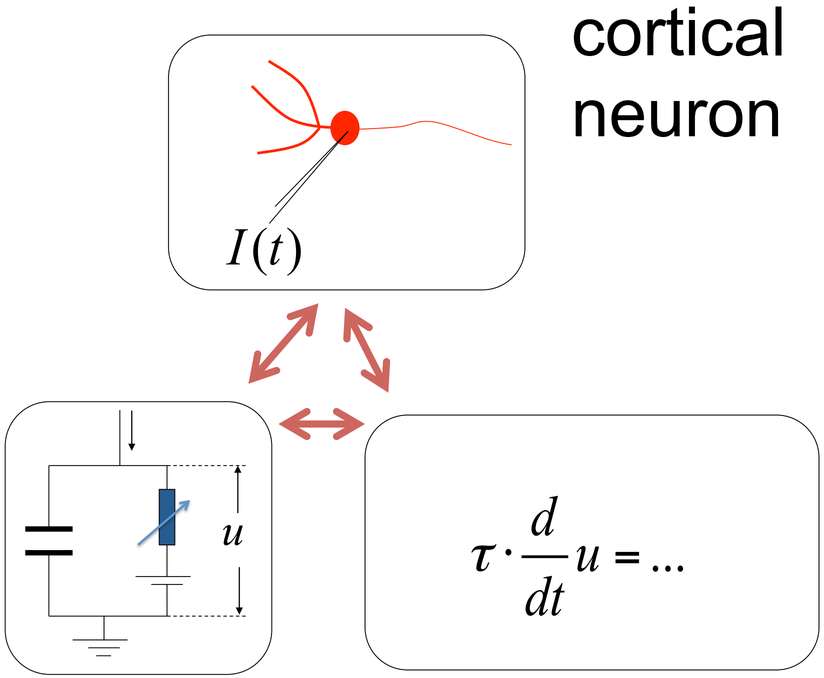
4.2 **Phase Plane Analysis**
- Role of nullclines

4.3 **Analysis of a 2D Neuron Model**
- MathDetour 3: Stability of fixed points

4.4 **TypeI and II Neuron Models**
- where is the firing threshold?
- separation of time scales

4.5. **Nonlinear Integrate-and-fire**
- from two to one dimension

Neuronal Dynamics – 4.1. Review :Hodgkin-Huxley Model



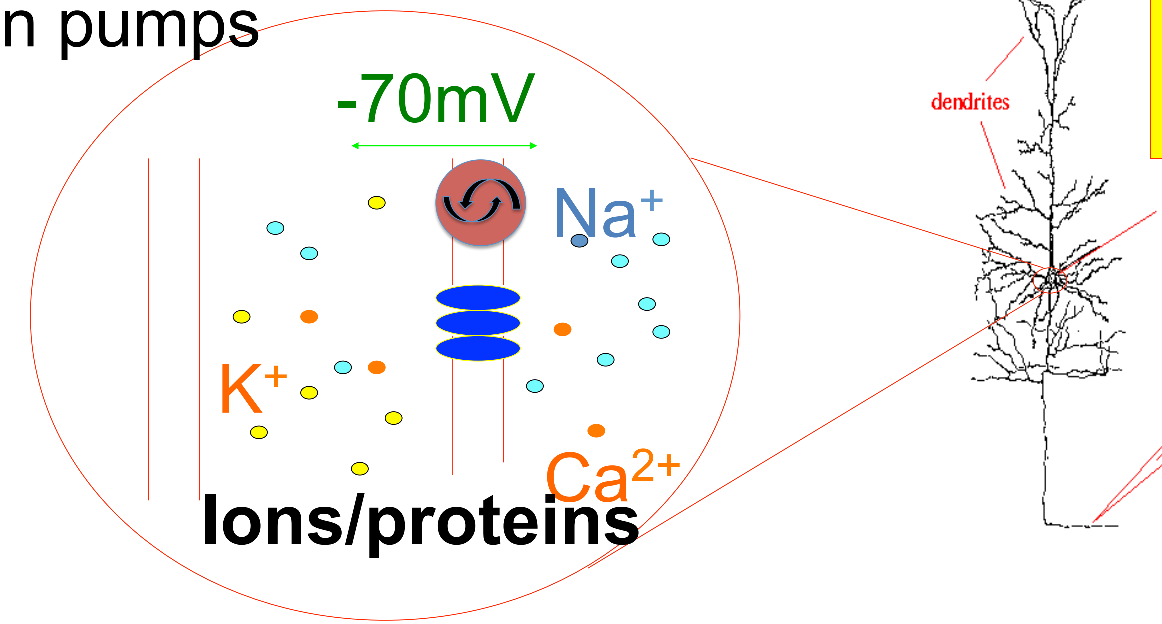
→Hodgkin-Huxley model
→Compartmental models

Neuronal Dynamics – 4.1 Review :Hodgkin-Huxley Model

Week 2:

Cell membrane contains

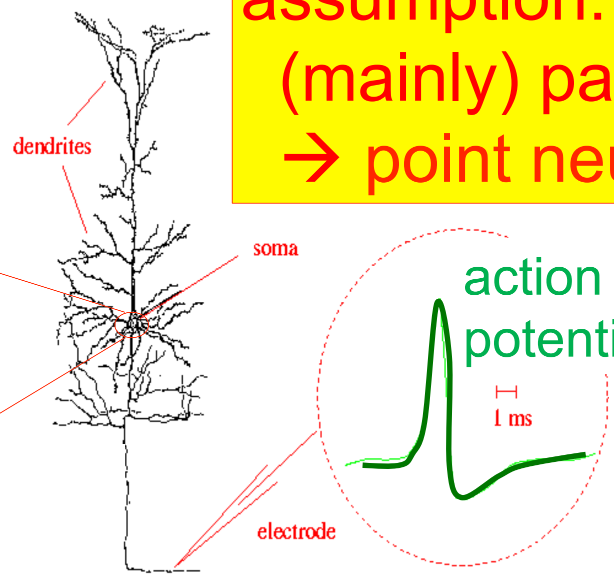
- ion channels
- ion pumps



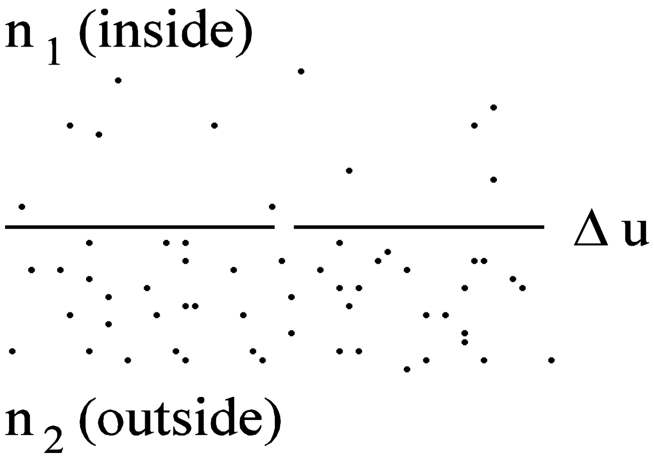
Dendrites (week 3):

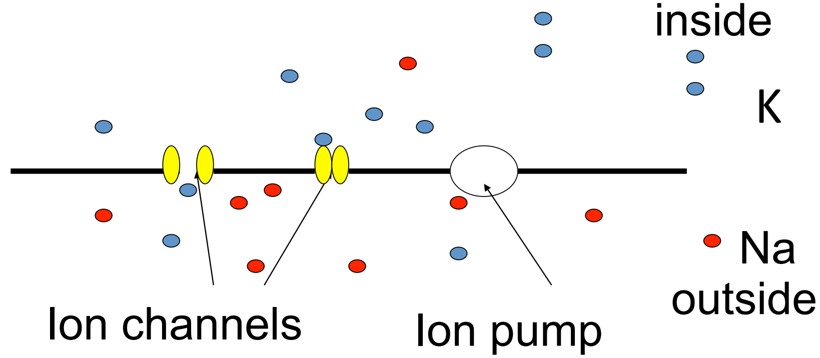
Active processes?

assumption:
(mainly) passive
→ point neuron



Neuronal Dynamics – 4.1. Review :Hodgkin-Huxley Model





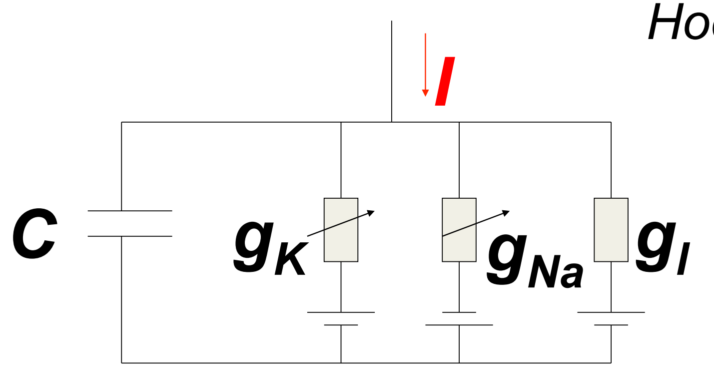
$$\Delta u = u_1 - u_2 = \frac{-kT}{q} \ln \frac{n(u_1)}{n(u_2)}$$

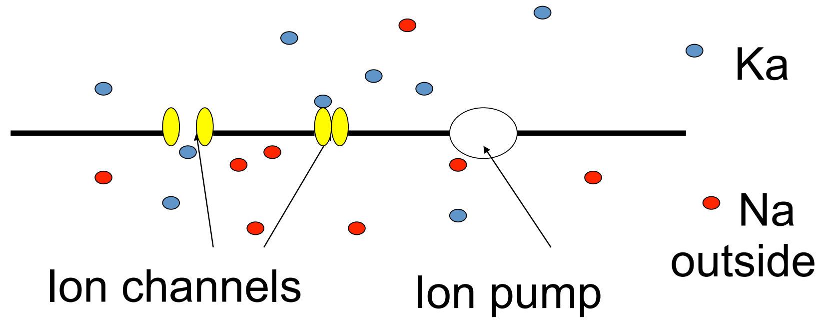
Reversal potential

ion pumps →concentration difference ⇔ voltage difference

Neuronal Dynamics – 4.1. Review: Hodgkin-Huxley Model

Hodgkin and Huxley, 1952





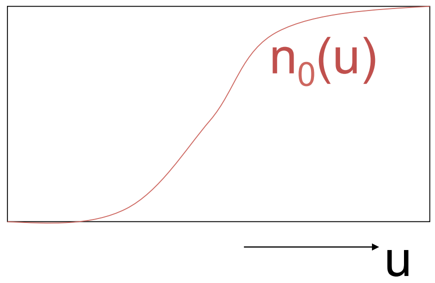
stimulus

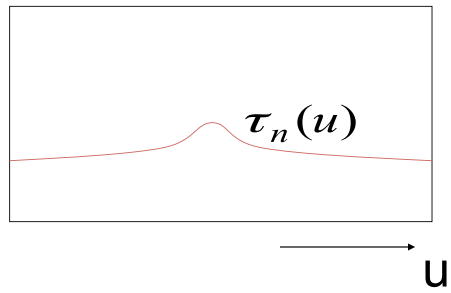
$$C\frac{du}{dt} = \overbrace{-g_{Na}m^3h(u-E_{Na})}^{I_{Na}} \overbrace{-g_K n^4(u-E_K)}^{I_K} \overbrace{-g_l(u-E_l)}^{I_{leak}} + I(t)$$

4 equations = 4D system

$$\frac{dm}{dt} = -\frac{m - m_0(u)}{\tau_m(u)}$$

$$\frac{dh}{dt} = -\frac{h - h_0(u)}{\tau_h(u)}$$





Neuronal Dynamics – 4.1. Overview and aims

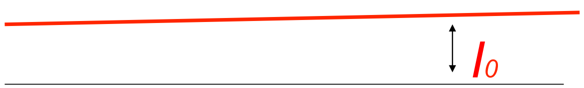
Can we understand the dynamics of the HH model?

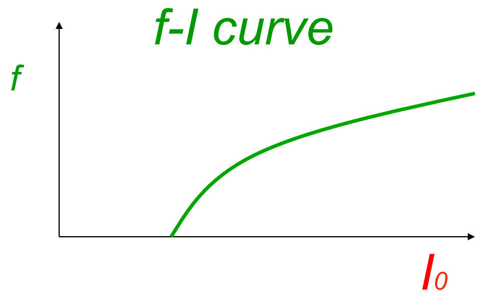
- mathematical principle of Action Potential generation?
- Types of neuron model (type I and II)?
- threshold behavior?

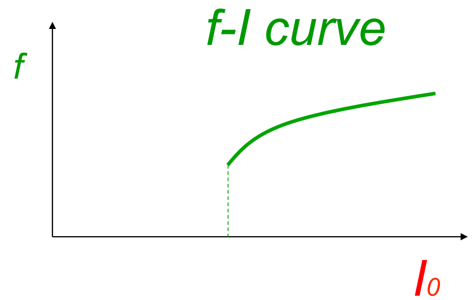
→ Reduce from 4 to 2 equations

Type I and type II models

ramp input/
constant input







Toward a two-dimensional neuron model

-**Reduction of Hodgkin-Huxley to 2 dimension**

-step 1: separation of time scales

-step 2: exploit similarities/correlations

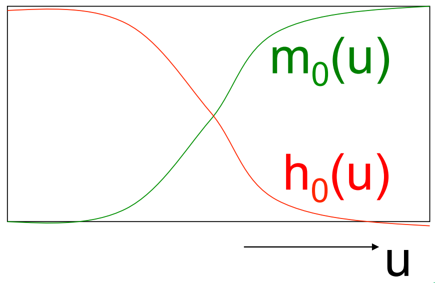
Neuronal Dynamics – 4.1. Reduction of Hodgkin-Huxley model

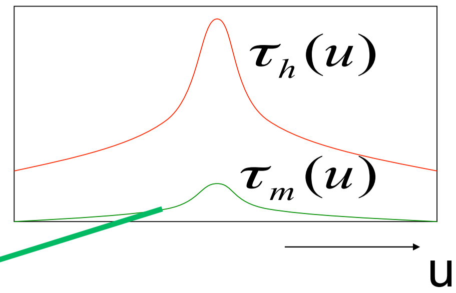
$$C\frac{du}{dt} = \overbrace{-g_{Na}m^3h(u-E_{Na})}^{I_{Na}} \overbrace{-g_K n^4(u-E_K)}^{I_K} \overbrace{-g_l(u-E_l)}^{I_{leak}} + \overset{\text{stimulus}}{I(t)}$$

$$\frac{dm}{dt} = -\frac{m-m_0(u)}{\tau_m(u)}$$

$$\frac{dh}{dt} = -\frac{h-h_0(u)}{\tau_h(u)}$$

$$\frac{dn}{dt} = -\frac{n-n_0(u)}{\tau_n(u)}$$





MathDetour 4.1

1) dynamics of *m* is fast $\longrightarrow$ $m(t) = m_0(u(t))$

Neuronal Dynamics – 4.1. Reduction of Hodgkin-Huxley model

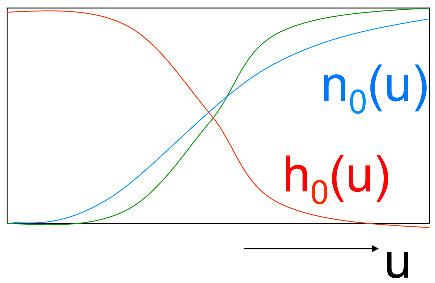
stimulus

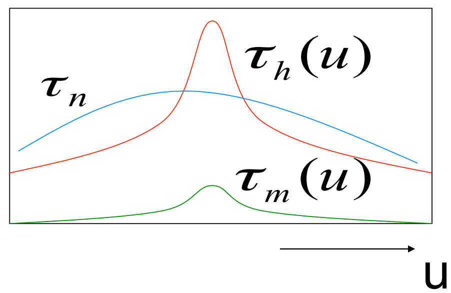
$$C\frac{du}{dt} = -\overbrace{g_{Na}m^3h(u-E_{Na})}^{I_{Na}} - \overbrace{g_K n^4(u-E_K)}^{I_K} - \overbrace{g_l(u-E_l)}^{I_{leak}} + I(t)$$

$$\frac{dm}{dt} = -\frac{m - m_0(u)}{\tau_m(u)}$$

$$\frac{dh}{dt} = -\frac{h - h_0(u)}{\tau_h(u)}$$

$$\frac{dn}{dt} = -\frac{n - n_0(u)}{\tau_n(u)}$$





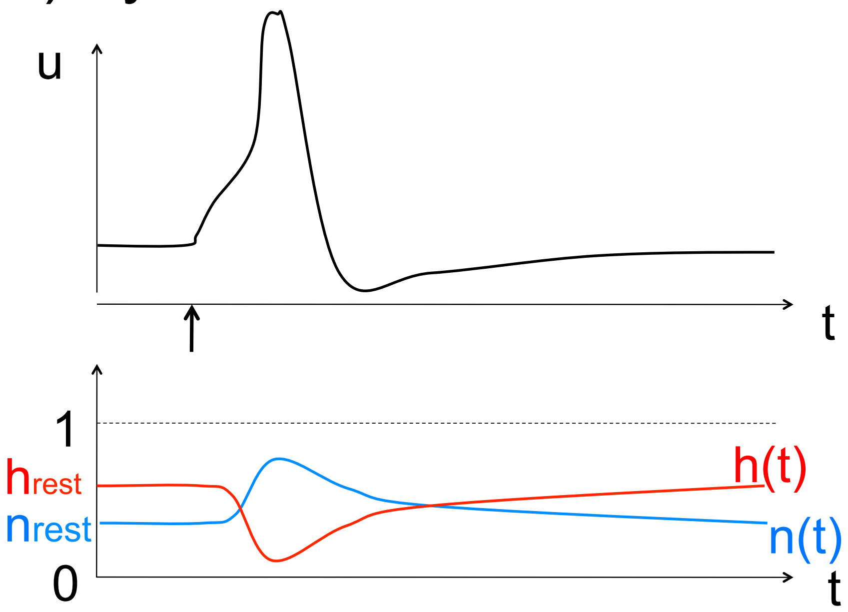
1) dynamics of *m* are fast $\longrightarrow$ $m(t) = m_0(u(t))$

2) dynamics of *h* and *n* are similar

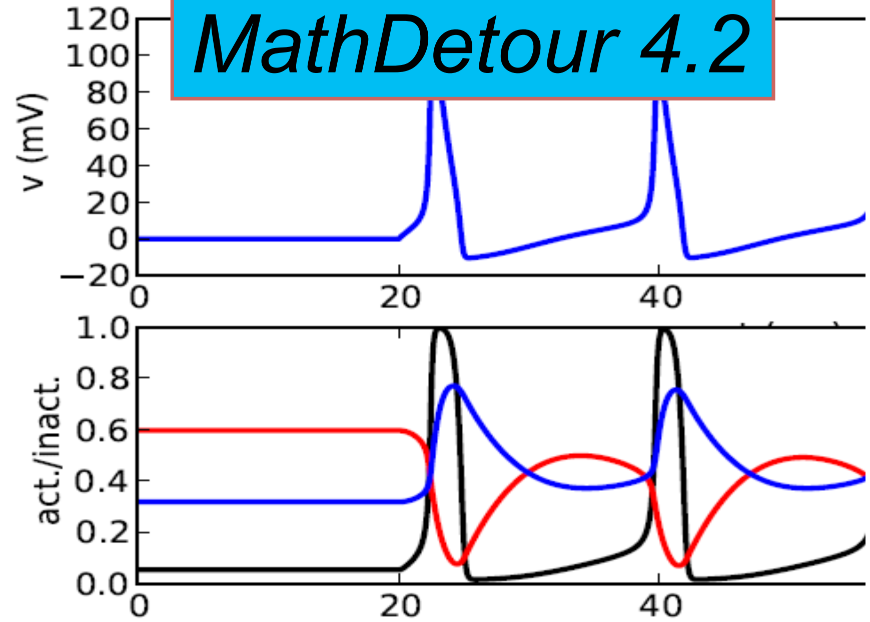
Neuronal Dynamics – 4.1. Reduction of Hodgkin-Huxley model

$$C\frac{du}{dt} = -g_{Na}m^3h(u-E_{Na}) - g_K n^4(u-E_K) - g_l(u-E_l) + \underbrace{I(t)}_{\text{stimulus}}$$

2) dynamics of *h* and *n* are similar $\longrightarrow$ $1 - h(t) = a\, n(t)$



MathDetour 4.2



Neuronal Dynamics – 4.1. Reduction of Hodgkin-Huxley model

$$C\frac{du}{dt} = \underbrace{-g_{Na}[m(t)]^3 h(t)(u(t)-E_{Na})}_{I_{Na}} \underbrace{-g_K[n(t)]^4(u(t)-E_K)}_{I_K} \underbrace{-g_l(u(t)-E_l)}_{I_{leak}} + I(t)$$

$$C\frac{du}{dt} = -g_{Na}\, m_0(u)^3(1-w)(u-E_{Na}) - g_K[\frac{w}{a}]^4(u-E_K) - g_l(u-E_l) + I(t)$$

1) dynamics of *m* are fast $\longrightarrow$ $m(t) = m_0(u(t))$

2) dynamics of *h* and *n* are similar $\longrightarrow$ $\underbrace{1-h(t)}_{w(t)} = \underbrace{a\,n(t)}_{w(t)}$

Neuronal Dynamics – 4.1. Reduction of Hodgkin-Huxley model

$$C\frac{du}{dt} = -\overbrace{g_{Na}\, m_0(u)^3(1-w)(u-E_{Na})}^{I_{Na}} - \overbrace{g_K\left(\frac{w}{a}\right)^4(u-E_K)}^{I_K} - \overbrace{g_l(u-E_l)}^{I_{leak}} + I(t)$$

$$\frac{dw}{dt} = -\frac{w-w_0(u)}{\tau_{eff}(u)}$$

$$C\frac{du}{dt} = f(u(t), w(t)) + I(t)$$

$$\frac{dw}{dt} = g(u(t), w(t))$$

Neuronal Dynamics – 4.1. Reduction to 2 dimensions

2-dimensional equation

$$C\frac{du}{dt} = f(u(t), w(t)) + I(t)$$

$$\frac{dw}{dt} = g(u(t), w(t))$$

Enables graphical analysis!

-Discussion of threshold
-Type I and II
- Repetitive firing

Neuronal Dynamics – Quiz 4.1.

A- Assumptions: In order to reduce a detailed compartmental neuron model to two dimensions we have to assume that

[ ] dendrites can be approximated as passive
[ ] the neuron model has no dendrite
[ ] the neuron model has at most 2 types of ion channels
[ ] all gating variables are fast
[ ] no gating variable is fast
[ ] gating variables fall in two groups: those that are fast and those that are slow
[ ] at least one of the ion channels is inactivating
[ ] the neuron does not generate spikes

B - A biophysical point model with 3 ion channels, each with activation and inactivation, has a total number of equations equal to

[ ] 3 or [ ] 4 or [ ] 6 or [ ] 7 ; [ ] 8 or more

C- Separation of time scales:

We start with two equations

$$\tau_1 \frac{dx}{dt} = -x + I(t)$$

$$\tau_2 \frac{dy}{dt} = -y + x^2 + A$$

We assume that $\tau_1 = \tau_2$

In this case a reduction of dimensionality

[ ] is not possible
[ ] is possible and the result is

$$\tau_2 \frac{dy}{dt} = -y + [I(t)]^2 + A$$

[ ] is possible and the result is

$$\tau_1 \frac{dx}{dt} = -x + x^2 + A$$