Introduction to Aerospace Structures and Materials

Dr. ir. R.C. (René) Alderliesten

Introduction to Aerospace Structures and Materials

Introduction to Aerospace Structures and Materials

R.C. Alderliesten

Delft University of Technology
Delft, The Netherlands

Cover image CC-BY TU Delft is a derivation of two images by: Christopher Boffoli, Big Appetites Studio, Seattle, Washington, USA, (http://bigappetites.net), who graciously agreed for us to use his Public Domain photograph of the Boeing 787 fuselage in high resolution, and by Gillian Saunders-Smits, Delft University of Technology with her photograph of a Fokker F100 Cockpit Structure (CC-BY-SA 4.0). The final cover design was made by Marco Neeleman, Delft University of Technology Library.

Partly funded by the TU Delft Extension School (online-learning.tudelft.nl) as part of the development of a Massive Open Online Course in Introduction to Aerospace Structures and Materials.
ISBN E-Pub: 978-94-6366-077-8
ISBN hardcopy: 978-94-6366-074-7
ISBN PDF: 978-94-6366-075-4

8. Considering strength & stiffness

8.1 Introduction

With the previous chapter in mind, one may understand material behaviour in general, one may know the structural elements of which aeronautical structures are made of, and one may even know about how loads can be transferred through a structure. However, this knowledge may still leave a gap of understanding what aspects are needed to be considered when designing structures and especially materializing them, i.e. selecting appropriate materials for the structure.

To give an impression of aspects to be considered when selecting a material and structural design for a particular application, this chapter will discuss and explain aspects and criteria that relate to material performance in a structural design with a focus on strength and stiffness.

8.2 Structural performance

The performance of a structure can be expressed in many different aspects. For example, one may consider the strength at which the structure ultimately fails an important measure for its performance, but if the most dominant requirement is whether the structure under certain loads is limited in deflection, then it may not be the most important performance aspect.

Especially for aeronautical structures weight is considered an important structural aspect. Lightweight design has become an expertise in itself, which seems to aim for structural designs with the lowest possible weight.

Here, one must pay attention to the header above this section, because when discussing weight as structural performance parameter, one should not confuse that with weight or density of structural materials. Although the density of a material will play a certain role in the final structural weight, both categories of weight do not directly relate. Or to put it differently, selecting a material with low density or weight, does not automatically lead to a structure with low weight.

Aside from manufacturing aspects to be considered, the structural performance is a function of

- Properties of materials used in the structural design
- Geometrical features and dimensional aspects of that particular structure

This is illustrated in Figure 8.1. One should be aware, however, that these two categories are not independent of each other.

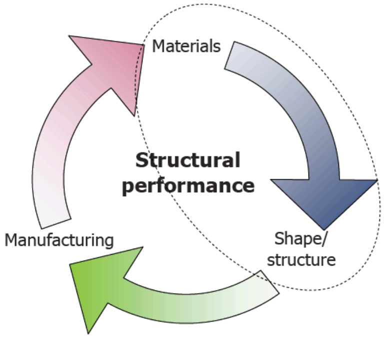


Figure 8.1
Illustration of the optimization considered in this chapter. (Alderliesten, 2011, 8-1.jpg. Own Work.)

For example, an all aluminium aircraft can be designed and manufactured in different ways, where the structural performance is a function of the aluminium alloy properties and the structural geometry. Although balsawood has a significant lower density (and thus weight) than aluminium, designing and constructing an aircraft from that material will lead in the end to a completely different design, in which the geometrical aspects most likely will counteract the benefit of the lower material weight.

Obviously, the strength of balsa wood is significant lower than aluminium which to some extent will play a role in the design, but it is obvious that other aspects like durability and environmental considerations (moisture absorption and subsequent reduction of properties) will also play a role in the selection of materials.

A statement that a certain material has been selected for its low material weight, as reason to obtain low structural weight, testifies therefore for not understanding the concept of structural performance. This will be demonstrated in this chapter with respect to strength and stiffness.

8.3 Selecting the appropriate criterion

The optimization of a structural design may lead to different kind of solutions depending on the parameters to which has been optimized. For example, for the same structural application one may obtain different design concepts and even made of different materials, if the concept is optimized for its lowest weight or for lowest manufacturing or operational cost.

But for clarity, the current chapter will primarily focus on the optimization with respect to weight, i.e. the identification of the relevant criteria for the evaluation of mechanical properties of materials. It has been mentioned before (chapter 3) that the strength-to-weight ratio of materials is often considered to select the best material for a structure.

8.3.1 Assessment of material or geometry

First of all, one has to be aware that the lowest weight to be achieved is structural weight and not material weight. This means that if one aims to optimize a structure to the lowest possible weight, not only the material properties and related density have to be assessed, but also the geometrical aspects. This is illustrated in Figure 8.2.

Although this seems to be straightforward, it often leads to confusion. With the introduction of another material in a certain application, the geometry or shape is sometimes also changed. The comparison between old and new design is then often directly related to the introduction of the new material. However, as may be understood from Figure 8.2, one has to distinct between the effect of shape and the effect of the material properties.

However, this distinction is not always easy to make. The example of the steel and aluminium bicycles in section 8.3.3 illustrates that the introduction of a new material is not always possible with the same geometry; small diameter tubes made of aluminium will not provide enough stiffness to the bicycle frame.

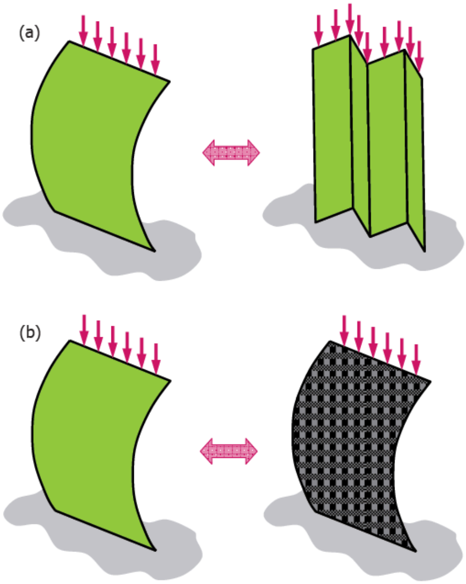


Figure 8.2
Evaluating geometrical aspects for the same material (a) and material aspects for the same geometry (b).
(Alderliesten, 2011, 8-2.jpg. Own Work.)

8.3.2 Specific tensile strength

The specific strength is usually defined as the ratio between strength σ_{ult} and density ρ. This ratio relates to another parameter often used in design and construction: the breaking length. The breaking length provides in fact a physical interpretation of the specific strength. Consider a hanging bar, illustrated in Figure 8.3 with a length L, a cross-section A, a failure strength σ_{ult} and a density ρ.

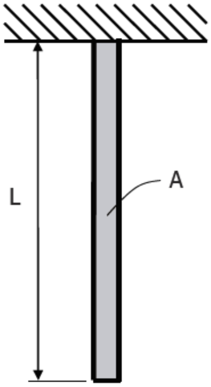


Figure 8.3
Hanging bar with cross-section A. (Alderliesten, 2011, 8-3.jpg. Own Work.)

The weight W in [N] is given by:

$$W = LA\rho g \tag{8.1}$$

where g is the gravitational acceleration. The stress in the upper cross-section is

$$\sigma = \frac{W}{A} = \frac{AL\rho g}{A} = L\rho g \tag{8.2}$$

The length of the bar in Figure 8.3 can be extended as far as the upper cross-section can carry the load. The length at which the upper cross-section reaches its failure strength is defined as the breaking length.

$$L_{ult} = \frac{\sigma_{ult}}{\rho g} \tag{8.3}$$

The unit for the breaking length is usually [km]. A correlation between several materials, their specific strength and the breaking length is given in Table 8.1.

Table 8.1
Correlation between specific strength and breaking length

Material	σ_{ult} [MPa]	σ_{ult}/ρ [10^6 Nmm/kg]	ρg [N/dm^3]	L_{ult} [km]
Steel AISI 301	1275	159	78.4	16.2
Steel D6AC	1931	248	77.2	25.0
Aluminium 2024-T3	483	174	27.3	17.7
Aluminium 7475-T761	517	184	27.6	18.7
Magnesium AZ31-H24	290	163	17.5	16.6
Titanium Ti-6Al-4V (5)	950	214	43.5	21.8
Quasi-isotropic CFRP	500	327	15.0	33.3

Example: Specific strength of a simple tension bar

Consider a tension bar to transfer load of 1000 kN from point A to point B over a length of 2 m. Which material (steel or aluminium) provides the lightest solution?

	Steel	Aluminium
Failure strength	800 N/mm^2	450 N/mm^2
Yield strength	550 N/mm^2	280 N/mm^2
Density	7.8 kg/dm^3	2.8 kg/dm^3

If permanent (plastic) deformation is not allowed, then the maximum allowed stress equals the yield strength of the materials. The minimum required cross-section is obtained by dividing the load of 1000 kN by the yield strength,

$$A_{steel} = \frac{P}{\sigma_{y,steel}} = \frac{1 \cdot 10^6}{550} = 1818 \text{mm}^2$$

and,

$$A_{alum} = \frac{P}{\sigma_{y,alum}} = \frac{1 \cdot 10^6}{280} = 3571 \text{mm}^2$$

The weight that corresponds to these bars is obtained by multiplying the volume with the density.

$$W_{steel} = \rho_{steel} A_{steel} L = 7.8 \cdot 0.1818 \cdot 20 = 28.4\text{kg}$$

$$W_{alum} = \rho_{alum} A_{alum} L = 2.8 \cdot 0.3571 \cdot 20 = 20.0\text{kg}$$

Thus the aluminium solution is 42% lighter than the steel solution.

This could have been evaluated alternatively by directly comparing the performance/weight ratio.

$$\frac{F}{W} = \frac{\sigma_y \cdot A}{\rho \cdot A \cdot L} = (\frac{\sigma_y}{\rho}) \cdot (\frac{1}{L})$$

Because the length L is equal for both cases (thus geometrical aspects are the same, see Figure 8.2), the ratio σ_y / ρ should be considered. This yields 70.5*10^6 Nmm/kg for steel and 100*10^6 Nmm/kg for aluminium. Thus aluminium has the highest specific strength for this case.

The correlation between specific strength and breaking length in Table 8.1 clearly does not provide the reason why aluminium is so often applied in aircraft structures. This is because the specific strength, or more precisely formulated, the specific tensile strength is not the only parameter determining the material to be applied. Depending on the application and the load cases, different parameters have to be considered.

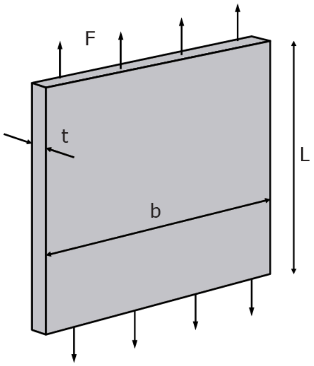


Figure 8.4
Thin sheet loaded in tension. (Alderliesten, 2011, 8-3.jpg. Own Work.)
Consider a thin sheet loaded in tension as illustrated in Figure 8.4 made of either AISI 301 steel or magnesium AZ31-H24 (see Table 8.1). The specific tensile properties of both materials are fairly close to each other. The specific modulus of elasticity for these alloys is respectively 2.5 and 2.4 Nmm kg^{-1}.

In particular, the structures applied in aircraft and spacecraft are considered to be thin-walled, i.e. the thickness is relatively small compared to the other dimensions. To illustrate the significance of the specific properties of materials in relation to the application in airframes, two metals are compared.

8.3.3 Specific buckling strength

Thus, concerning the tensile load applied to the sheet, there is no preference for either one of the two metals. However, if compression is considered as load case, sheet buckling has to be considered. The critical buckling strength of a sheet is given by

$$\sigma_{cr} = \frac{Et^2}{Lb} \tag{8.4}$$

The buckling load is then defined as

$$F_{cr} = \sigma_{cr} bt \tag{8.5}$$

Thus, the required thickness is calculated with

$$t = \sqrt[3]{\frac{F_{cr}L}{E}} \tag{8.6}$$

Keeping the buckling load and panel length identical for both metals it is evident that the thickness of the magnesium AZ31-H24 is about 1.6 times the thickness required for AISI301 steel. However, this implies that the weight of the magnesium sheet is about 2.8 times lower than the steel sheet.

In other words, to compare the specific properties for the case of compression buckling, one has to search the highest value for $\sqrt[3]{E}/\rho$. Table 8.2 gives the values for this parameter for the materials in Table 8.1

Table 8.2
Specific buckling strength parameter

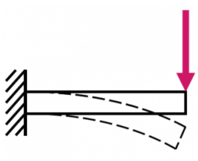
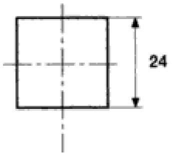
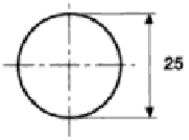
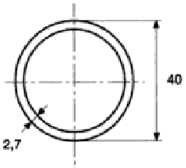
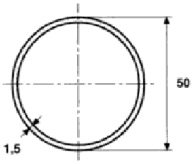
Material	σ_{ult} [MPa]	E [GPa]	$\sqrt[3]{E}/\rho$ [10^6 mm$^{7/3}$N$^{2/3}$]
Steel AISI 301	1275	193	0.72
Steel D6AC	1931	210	0.76
Aluminium 2024-T3	483	72	1.50
Aluminium 7475-T761	517	70	1.47
Magnesium AZ31-H24	290	45	2.00
Titanium Ti-6Al-4V (5)	950	114	1.09
Quasi-isotropic CFRP	500	60	2.56

8.4 Geometrical aspects

The comparison of material properties or, as discussed in the previous section, the specific material properties may indicate the applicability of certain materials for specific structural configurations and load cases. However, one has to be aware that one still has the opportunity to tailor the structure in its geometry, as discussed in section 8.3.1.

To illustrate this aspect, Table 8.3 compares four cross-sections for a beam loaded in bending. The comparison shows that significant weight reductions can be achieved by changing the cross section of the beam, while keeping the material the same. In fact, this is the reason why the aluminium bicycle frames are made of tubes with a larger diameter than the conventional steel frames, see the example in this section.

Table 8.3
Correlation of cross-sectional weight for equal bending stiffness

				
Weight	100%	81.7%	51.7%	20%

In Table 8.4 it is illustrated that when comparing several materials for their specific stiffness that these materials do not always rank highest for all cases. For example, bone material has a lower specific stiffness compared to aluminium, but when the sheet stability is considered, the value for aluminium is lower. This illustrates that the comparison of the materials should account not only for the geometry, but also for the representative loading scenario.

Table 8.4
Illustration of typical specific material properties.
(1) Represents weight % of similar loaded structure compared to steel.

Material	Specific modulus		Column stability		Sheet stability	
	E/γ	$\%^{(1)}$	$\sqrt{E}/\gamma$	$\%^{(1)}$	$\sqrt[3]{E}/\gamma$	$\%^{(1)}$
Aluminium	2500	108	9.5	62	1.5	52
Steel	2692	100	5.9	100	0.8	100
Spruce	2340	115	22.3	26	4.7	16
Birch	2538	106	19.8	30	3.9	19
Bone	1500	179	9.0	65	1.6	47
Titanium	2622	103	7.8	76	1.1	73
Isotropic carbon fibre composite	3333	81	14.9	40	2.5	32
Isotropic E-glass fibre composite	536	502	5.1	116	1.1	73
Isotropic aramid fibre composite	1760	153	11.4	52	2.1	38

Example: Steel and aluminium bicycle frames

For many years bicycle frames were made of steel tubes with circular cross section. These tubes were jointed by either lugs or welding. Because of the high strength and stiffness of the steel alloys used, tubes could be used with a relatively small circular cross section. The small circular tubes could be joined easily with lugs, in which the tubes were then brazed to the lug. The alternative joining method is TIG welding which is a straightforward process for steels.

Aluminium, although having better strength-to-weight ratio than steel was not applied, because the applied aluminium alloys could not be welded. With the introduction of weldable aluminium alloys, the introduction of aluminium frames was initially not successful because of fatigue failures. Aluminium usually has a lower fatigue limit than steel, see explanation in chapter 10.

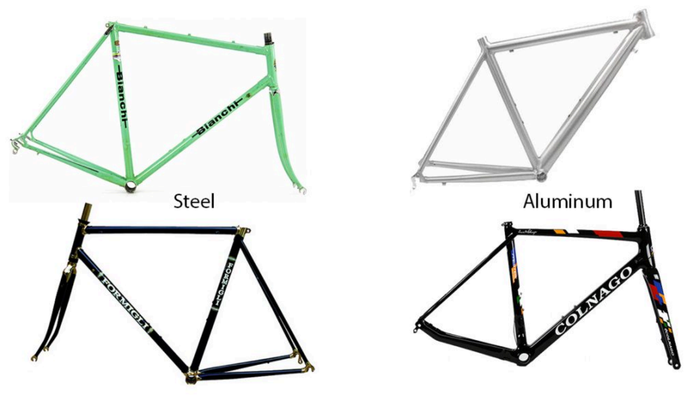


Figure 8.5
Comparison between steel and aluminium bicycle frames. Frame top left: Steel-vintage.com, (2018), CC-BY; frame top right: fietstijden.nl, (2018), Public Domain; bottom left: Saunders, (2008), CC-BY-NC-ND2.0; bottom right: Glory cycles, (2018), CC-BY2.0.

Once reliable and weldable aluminium frames were introduced, many aluminium bikes were sold for their low weight compared to steel bikes. Although the aluminium has a lower stiffness than steel., rigid frames could be achieved by changing the cross-sectional area and shape compared to the steel tubes. Aluminium frames are therefore easily recognized because of the larger tube cross-sections, see Figure 8.5.

8.5 Structural aspects

The geometrical aspects discussed in the previous section are important to be considered. The transition in using different materials often comes together with the application of different design concepts. An illustration is provided in Figure 8.6, where the wood, linen and steel trusses of the early aircraft are compared with the stiffened aluminium shell structure of current commercial aircraft. The concept of load bearing shell structures could only be applied when materials were considered that can be loaded as such.

Figure 8.6
Illustration of the different design related to material usage; wood, linnen and steel trusses (left) and load bearing stiffened aluminium shell structure. Derivative from left: Cliff, (2003), CC-BY2.0 and right: Kolossos, (2006), CC-BY-SA3.0.

Although this seems a very straightforward aspect, often the comparison of material technologies is performed solely by addressing the material properties. For example, the fact that carbon composites in general have lower densities than aerospace aluminium alloys is used to explain that these fibre reinforced materials will result in lighter structures.

However, even if similar structures are being considered – for example load bearing shell structures – still the comparison may need attention. Comparing aluminium with carbon fibre composites in a load bearing shell structure, will not automatically lead to similar details design solutions. Where aluminium stringers for example can easily be extruded into preferred shapes, the manufacturing technology for composites may require stiffeners with different geometries.

In addition, the difference between aluminium and carbon fibre composites may lead to different selection of shell concept, i.e. shell containing stiffeners and sandwich panel (discussed in chapter 5). The reasons for selecting either one of the two may be completely different for both material technologies.

But even when these aspects are considered, one has to be careful with comparing the materials purely on material properties as determined in a material test. The question here may be what strength of the material should be considered when comparing different technologies. Comparison based on ultimate strength, i.e. σ_{ult}, often leads to irrelevant weight estimates, because this parameter is in general not directly decisive for design.

Aluminium structure are required not to permanently deform under the maximum occurring loads, which implies that the yield strength often dictates the minimum thicknesses rather than the ultimate strength.

The restriction on strength that can be exploited in composites is even more severe, the maximum allowable strain that may occur in the structure is limited at about 0.35%. This implies a significant reduction in strength to design with.

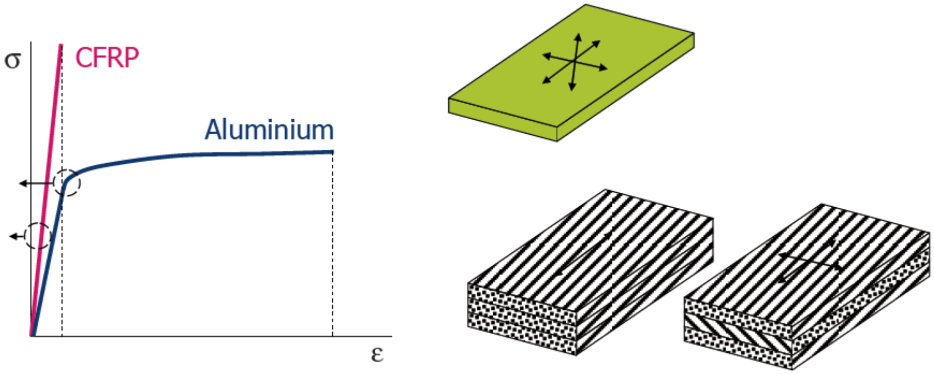


Figure 8.7
Comparison of strength values for different materials (left); high strength linear elastic composites may be limited relatively more than ductile aluminium. The aluminium properties relate to isotropic behaviour, while composite properties often relate to uni-directional composites (right).(Alderliesten, 2011, 8-7.jpg. Own Work.)

Comparing aluminium with carbon fibre composites therefore should be based on equivalent static or fatigue loads. As illustrated here, the static strength comparison may be based on the yield strength in aluminium and about a third of the panel strength (i.e. lay-up of individual plies in the required orientations, not the uni-axial ply strength, see Figure 8.7) in composites.

The actual composite panel lay-up that must be considered depends on the application one has in mind to compare the materials for. As illustrated in Figure 8.8, the composite panel lay-up for a vertical tail will be different from the lay-up needed for fuselage shell panels. But even there, the orientation in the upper fuselage (loaded primarily in tension) may be expected to be different form the side shells (loaded primarily in shear). Some typical values for comparison are provided in Table 8.5.

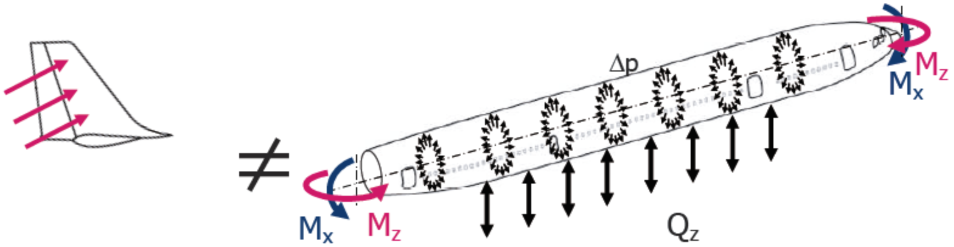


Figure 8.8

The panel lay-up for composite materials is different for the various applications on an aircraft; where empennage structures often allow significant directionality of composites leading to large weight savings, fuselage structures require almost quasi-isotropic laminate lay-ups. (TU Delft, n.d., 8-8.jpg. Own Work.)

Table 8.5
Comparison between the weight of different cross sections optimized for equal bending stiffness

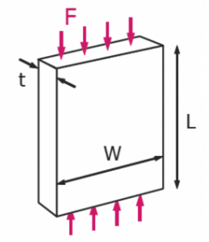
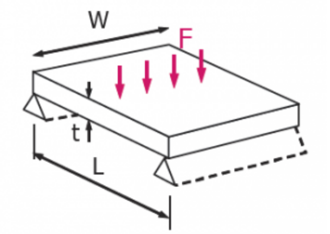
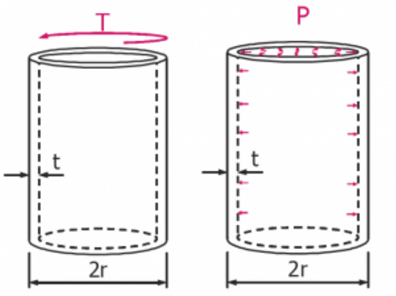
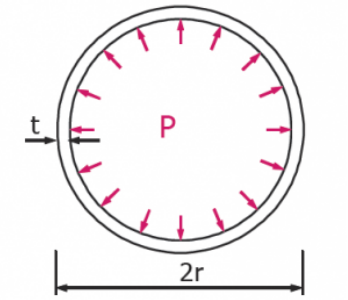
Loading mode	Minimum weight for given	
	stiffness	strength
	$\frac{\sqrt[3]{E}}{\rho}$	
	$\frac{\sqrt[3]{E}}{\rho}$	$\frac{\sqrt{q}}{\rho}$
	$\frac{E}{\rho}$	$\frac{\sigma}{\rho}$
	$\frac{E}{(1-\nu)\rho}$	

Table 8.6
Comparison between composites and aluminium based on relevant strength values, with (1) σ_{max} is based on σ_{ult} limited to 0.35% design strain and (2) σ_{max} is taken from σ_{ult} including notch factor 0.9 as specified in Rice et al., (2003).

Material	Lay-up % 0°/±45°/90°	E [GPa]	$\frac{E}{\rho}$ [MPa m^3/kg]	σ_{max} [MPa]	$\frac{\sigma_{max}}{\rho}$ [MPa m^3/kg]	Comment
CFRP (T800S)	60 / 30 / 10	103	64	360$^{(1)}$	0.225	Tail plane shells
CFRP (T800S)	40 / 50 / 10	77	48	270$^{(1)}$	0.170	
CFRP (T800S)	20 / 70 / 10	50	31	175$^{(1)}$	0.110	Fuselage side shell
2xxx Al-alloy		72	26	440$^{(2)}$	0.160	
7xxx Al-alloy		72	26	565$^{(2)}$	0.205	
Al-Li alloy		77	29	515$^{(2)}$	0.195	

8.6 Typical mission requirements for space structures

The design of the structural components in space structures usually starts with the determination of the mission requirements. However, when it comes to the typical missions for spacecraft and launchers, it is obvious that in general the mass should be minimized as much as possible, while the stiffness and the strength should be as high as possible.

In addition, for the launchers yields that they should be able to accommodate the payload and the equipment, and that their mission should be fulfilled with high reliability. The costs of a launching failure are extremely high (estimated at about 180 million Euros for the Ariane 5).

In general, these high demands to the structure imply that structural solutions and materials are often considered that are too expensive and complex for aeronautical structures. Nonetheless, even for spacecraft and launch vehicle structures the requirements are to reduce costs, and to search for the design solution which has proven to have the best manufacturability and accessibility.

8.7 Material selection criteria

The selection of materials for space applications is an important topic. The stiffness of the structure, and therefore also the materials used, is an important parameter to design against the resonance that may occur during launch of the vehicle.

Oscillations can be either damped or excited, see for example Figure 8.9. These oscillations or vibrations are very important in space structures, because during launch significant vibrations may occur. Examples of vibration problems in aeronautical applications are flutter of main and tail wings.

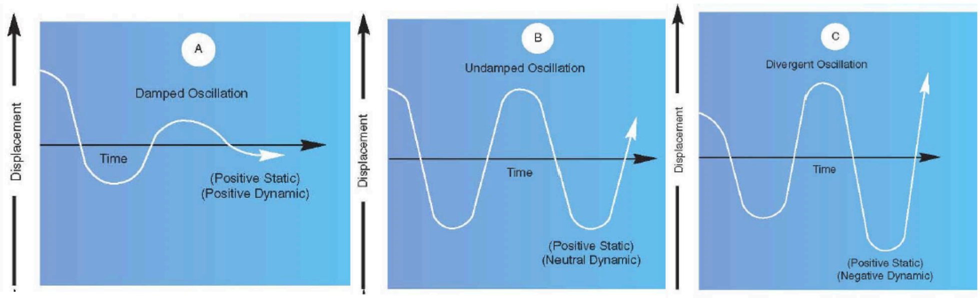


Figure 8.9
Illustration of the oscillations that are damped (left), not damped (centre) and divergent (right). (TU Delft, n.d., 8-9.jpg. Own Work.)

Therefore, limiting the natural frequencies of spacecraft is essential to avoid resonance between launch vehicle and spacecraft. In general low dynamic coupling results in lower loads for spacecraft. To dimension the primary structure of a spacecraft (such as a satellite for instance), the first step therefore is to assure that the lowest natural frequency present in the space craft structure is well above the specified minimum frequency by the launcher's user manual (Wertz & Larson, 1999). Once this has been achieved, the structure will be further designed and tailored for all the quasi-static loads that will occur. In this order these steps will be discussed in the following sections.

8.8 Structural sizing for natural frequency

The discussion of structural sizing for natural frequencies, will be explained here using simple examples. Consider the concentrated mass at the end of a clamped beam as illustrated in Figure 8.10. If resonance may occur in both axial and lateral direction, i.e. in x- and y-direction, then loading of the beam may be considered respectively by axial loading of a spring (see introduction of chapter 1) and bending of a beam.

For both cases a 'spring constant' k may be determined. For axial direction, the constant k is a function of the stiffness of the spring (represented by EA) and the length of the spring, as is illustrated by Figure 8.12.

$$k_x = \frac{EA}{L} \tag{8.7}$$

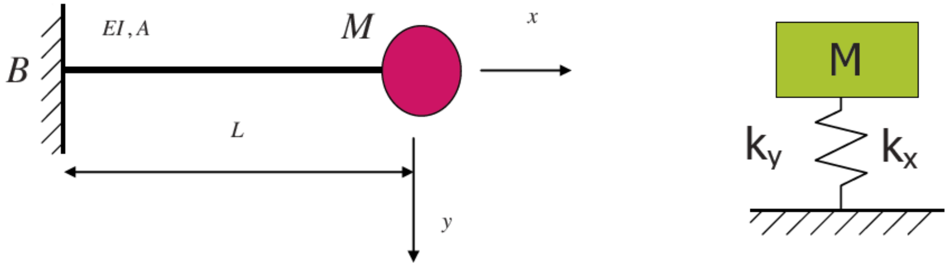


Figure 8.10
Schematic of a concentrated mass at the end of a clamped beam (left) and the simplification of the case to a single degree of freedom case (right). (Alderliesten, 2011, 8-10.jpg. Own Work.)

In lateral direction, the constant k is related to the bending stiffness of the spring (represented by EI) and the length of the spring

$$k_y = \frac{3EI}{L^3} \tag{8.8}$$

The axially loaded configuration, described by equation (8.7), can therefore be related to the case illustrated in Figure 8.11. According to chapter 1, the elongation and strain for this case can be described by

$$\Delta L = P\frac{L}{EA}, \ \varepsilon = \frac{\Delta L}{L} = \frac{P}{EA} = \frac{\sigma}{E} \tag{8.9}$$

Similarly, the deflection for the laterally loaded case can be given by

$$\delta = \frac{PL^3}{3EI} \tag{8.10}$$

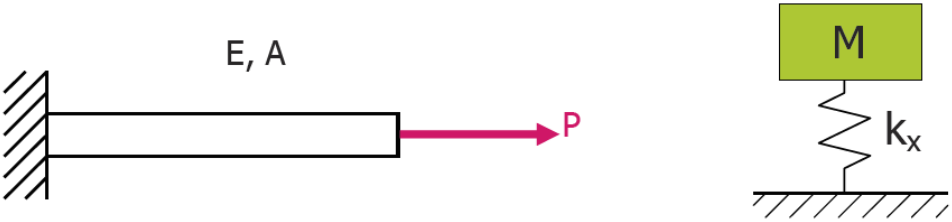


Figure 8.11
Illustration of the beam loaded in axial direction (left) and the simplification of the case to a single degree of freedom case (right). (Alderliesten, 2011, 8-11.jpg. Own Work.)

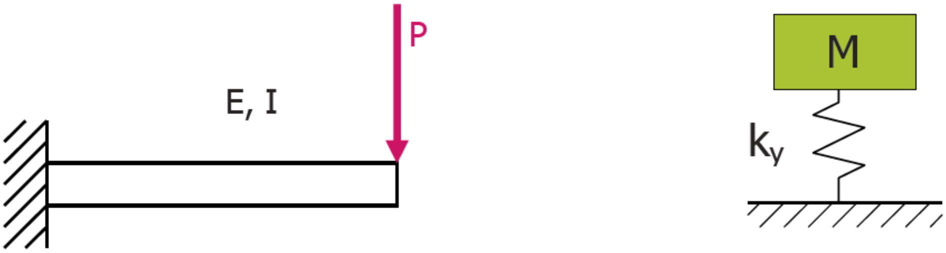


Figure 8.12
Illustration of the beam loaded in lateral direction (left) and the simplification of the case to a single degree of freedom case (right). (Alderliesten, 2011, 8-12.jpg. Own Work.)

The natural frequency in [Hz] is defined as

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{8.11}$$

where k is the spring constant and m the mass. If this equation represents the lowest natural frequency that is allowed in the structure, this means for the axial direction that

$$\frac{EA}{L} \geq (2\pi f_n)^2 m \tag{8.12}$$

and for the lateral direction that

$$\frac{3EI}{L^3} \geq (2\pi f_n)^2 m \tag{8.13}$$

8.9 Structural sizing for quasi-static loads

Once the requirements concerning natural frequency are being met, the structure can be designed for quasi-static loads. These loads are directly related to the acceleration of the mass during launch and ascent.

In axial direction, the load is given by the acceleration of the mass in axial (launch) direction with

$$F = mg_x \tag{8.14}$$

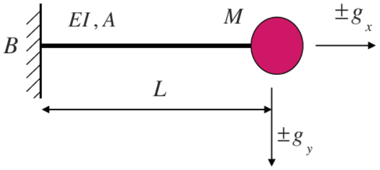


Figure 8.13
Schematic of a concentrated mass at the end of a clamped beam subjected to axial and lateral accelerations. (Alderliesten, 2011, 8-13.jpg. Own Work.)

With the cross section of A this gives in axial direction

$$\sigma_x = \frac{F}{A} \tag{8.15}$$

For the lateral loads, the bending moment applied by the mass is considered:

$$\sigma_x = \frac{My}{I} \tag{8.16}$$

Where I is the area moment of inertia for bending, y the distance from the neutral line to the outer surface of the beam, and M the moment given by:

$$M = FL = mg_x L \tag{8.17}$$

The allowable stress is the maximum stress that the structure should be capable to sustain without any damage or failure. This maximum stress is calculated by superimposing the stresses due to axial and lateral accelerations, which should be lower than the allowable stress.

$$\sigma_{tot} = \sigma_x + \sigma_y \leq \sigma_{allowable} \tag{8.18}$$

The allowable stress is the ultimate stress divided by a safety factor.

Another load case that has to be considered is the buckling load applied to the structure by the axial accelerations during ascend. In this case the bending stiffness EI of the beam becomes an important parameter. The Euler buckling load can be calculated with

$$F_{euler} = \frac{\pi^2 EI}{4L^2} \tag{8.19}$$

which implies that the loads due to axial accelerations should be limited to:

$$g_x M \leq F_{euler} \tag{8.20}$$

More on initial sizing of spacecraft can be found in Wertz & Larson (1999).

Example: Calculating the wall thickness of a satellite launched on ARIANE 5

Consider the ARIANE 5 launch vehicle illustrated in Figure 8.14. The Launch Vehicle User Guide for the ARIANE 5 lists the following requirements on the minimum natural frequencies that a payload that is send into space by this launch vehicle must meet: In longitudinal (launch) direction $f_1 > 31Hz$ and in lateral direction $f_2 > 10Hz$.

The following information on the satellite that is being launched is given:

- Payload mass, $m = 2500\,kg$
- Structure made of aluminium with $E_{al} = 72\,GPa$

- Maximum axial acceleration is $6g$ (launch load)
- Maximum lateral acceleration is $1.5\,g$ with $g = 9.81\ m/s^2$

The satellite may be modelled as a cylinder with dimensions as listed in Figure 8.14 with its entire mass modelled as a point mass on top. The structural mass of the cylinder may be ignored. This type of modelling is suitable as most satellites consist of some sort of cylinder (known as a “bus”) with most of the mass, the payload of the satellite itself, on top.

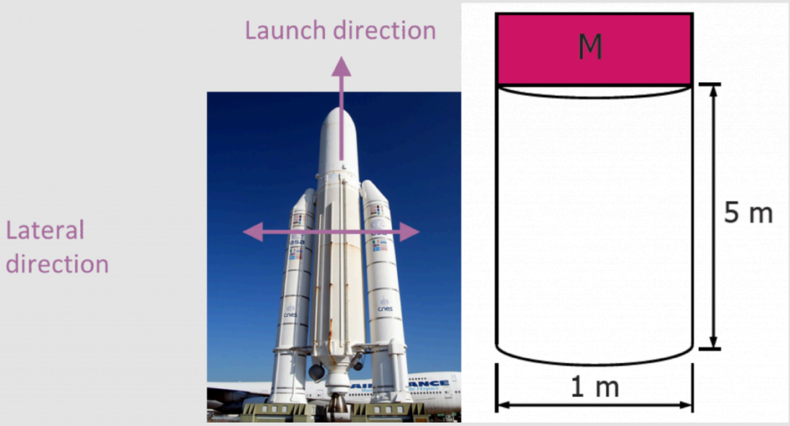


Figure 8.14

Illustration of the ARIANE 5 launch vehicle and the schematisation of the problem. Derivative from left: Turner, (2012), CC-BY-SA2.0 and Alderliesten, (2011), 8-14-a.jpg. Own Work.

What is the minimum required wall thickness of the structure? To find the answer to this we must calculate the required thickness based on both frequency requirements and select the larger of the two thicknesses found.

In launch direction, the natural frequency should comply with:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \geq f_1 = 31Hz$$

where m=2500 kg and k is given by

$$k = \frac{EA}{L}$$

This implies for A that:

$$A \geq \frac{mL}{E}(2\pi f_1)^2$$

With $A = 2\pi Rt$ this means that the minimum thickness required to meet the natural frequency requirement in launch direction: $t_1 = 2.1\,mm$.

In lateral direction, the natural frequency must be:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \geq f_2 = 10Hz$$

where again m = 2500 kg but the lateral stiffness k is given by:

$$k = \frac{3EI}{L^3}$$

This implies for the area moment of inertia I that:

$$I \geq (2\pi f_2)^2 \frac{mL^3}{3E}$$

using:

$$I = \pi R^3 t$$

results in a minimum required wall thickness to meet the natural frequency requirement in lateral direction, $t_2 = 15\,mm$.

The larger of the two calculated thickness is governing as that is the lower limit at which both requirements are met. Hence, $t_{min} = 15\,mm$.

With the thickness determined for the lowest natural frequency, the quasi-static loads can now be evaluated, starting with the allowable stress requirement.

$$\sigma_x = \frac{g_x m}{A} = \frac{F}{A}$$

where m=2500 kg, $g_x = 6g$ and $A = 2\pi rt = 47124\,mm^2$.
The axial stress is thus $\sigma_x = 3.1\,MPa$.

Now to calculate the lateral stress:

$$\sigma_y = \frac{g_y mLy}{I} = \frac{FLy}{I}$$

The area moment of inertia I for a thin walled cylinder is given by:

$$I = \pi R^3 t$$

resulting in $I = 5.9 \cdot 10^9\ mm^4$ The lateral acceleration was $1.5g$ and the maximum distance $y = 500\ mm$. The lateral stress is thus $\sigma_y = 15.6\ MPa$.

The total stress is the sum of the two:

$$\sigma_{tot} = \sigma_x + \sigma_y \leq \sigma_{allowable}$$

which is $\sigma_{tot} = 18.7\ MPa$. Of course one has to consider that $\sigma_{allowable} = \sigma_{ultimate}$ divided by the safety factor. With $\sigma_{ultimate} = 483\ MPa$ and a safety factor n = 2, $\sigma_{allowable} = 242\ MPa$ which is much greater than the total stress found, so the first quasi static load requirement is met.

To evaluate whether the second quasi-static load requirement is also met:

$$F_x \geq F_{Euler} = \frac{\pi^2 EI}{4L^2}$$

The load in launch direction is $6g$ which makes that $F_x = 0.15\ MN$ and $F_{Euler} = 41\ MN$, which means that the second quasi static load requirement has also been met.

Introduction to Aerospace Structures and Materials

Dr.ir. R.C. (René) Alderliesten

This book provides an introduction to the discipline of aerospace structures and materials. It is the first book to date that includes all relevant aspects of this discipline within a single monologue. These aspects range from materials, manufacturing and processing techniques, to structures, design principles and structural performance, including aspects like durability and safety. With the purpose of introducing students into the basics of the entire discipline, the book presents the subjects broadly and loosely connected, adopting either a formal description or an informal walk around type of presentation. A key lessons conveyed within this book is the interplay between the exact science and engineering topics, like solid material physics and structural analysis, and the soft topics that are not easily captured by equations and formulas. Safety, manufacturability, availability and costing are some of these topics that are presented in this book to explain decisions and design solutions within this discipline.

Dr.ir. R.C. (René) Alderliesten
TU Delft | Faculty of Aerospace Engeneering

Dr. Alderliesten obtained his MSc and PhD degree both at TU Delft, and holds since 2012 the position of associated professor within the department of Aerospace Structures and Materials at the faculty of Aerospace Engineering, TU Delft. His expertise is fatigue and damage tolerance of metals, composites and hybrid materials, with the emphasis on proper understanding the physics of damage growth. Dr. Alderliesten introduces Aerospace Structures & Materials in the first semester of the BSc curriculum, while teaching Fatigue of Structures & Materials in the first semester of the MSc both at TU Delft and at the University di Bologna.

ISBN 978-94-6366-075-4
DOI https://doi.org/10.5074/t.2018.003

textbooks.open.tudelft.nl

Cover image is licensed under CC-BY TU Delft is a derivative of images by: Christopher Boffoli, USA, of the 787 fuselage (CC-BY-SA 3.0), and Gillian Saunders-Smits, TU Delft of a Fokker F100 cockpit (CC-BY-SA 3.0).