

**SPEAKER:** The next case is the case where we know, two sides, and they include an angle. In these example, or in this drawing, we know sides A and B, And they include an angle gamma. And we are asked to find the remaining information for the triangle.

Here, we cannot use the sum of all the angles being 118 degrees because we have two unknown angles, alpha and beta. So let us see which law to use. Shall we use the law of sines? Or should we use the law of cosines?

Let us write down the law of cosines for the known side C. The law of cosines for the unknown side C states that C squared will be equal to A squared plus B squared minus 2 times the sides B and A times the cosine of the included angle gamma.

So if we carefully observe this expression, we observe that the right-hand side contains known information, which means that if I have dividers now, I can evaluate the numeric result. So at this stage, what do we know? We know all the sides and one of the angles. If we know this information, can we evaluate the other two angles? Well, we can apply now the law of sines because we know the sides and an angle. So let us apply the law of sines.

If we apply the law of sines, we observe that we can use these two equalities, this equality, these two fractions. And from here we can solve for the sine of the unknown angle alpha. So this is A divided by sine of alpha is equal to C divided by sine of gamma. And from here, we can solve for sine of alpha.  $s A/C \text{ times sine of gamma.}$

Then we can use the other equality, and we can solve for sine of beta. So I'm going to skip this step here and leave it as an exercise for you to write the equalities. And then here, we solve for sine of beta as  $B/C \text{ times sine of gamma.}$

So the last two expressions that gives us the sine of beta. And the sine of alpha could be evaluated given the fact that we know angle gamma. We know all the sides A and B. Of course, on could evaluate the angles using also the law of cosines if we

arrive at this stage, where we know one of the angles and all of the sides.

So let us write the expression that will allow us to evaluate, let's say, angle alpha. So we will write the law of cosines for angle alpha. These will be  $A^2$  is equal to  $B^2$  plus  $C^2$  minus 2 times  $B$  times  $C$  times cosine of angle alpha.

And of course now, we can manipulate this equation. You can rearrange it as cosine of alpha is equal to  $A^2$  minus  $B^2$  minus  $C^2$  divided by 2 times  $B$  times  $C$ . And if we have the values for the different sides, then we can solve these, finding the numeric result for the fraction, and then use our calculator to solve the inverse cosine. And find the numeric answer for alpha.