Week 6 – part 2 : Interspike intervals and renewal processes

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Neuronal Dynamics:
Computational Neuroscience of Single Neurons

Week 6 – Noise models:
Escape noise

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6.1 Escape noise
- stochastic intensity and point process

6.2 Interspike interval distribution
- Time-dependend renewal process
- Firing probability in discrete time

6.3 Likelihood of a spike train
- likelihood function

6.4 Comparison of noise models
- escape noise vs. diffusive noise

6.5. Rate code vs. Temporal Code
- timing codes
- stochastic resonance

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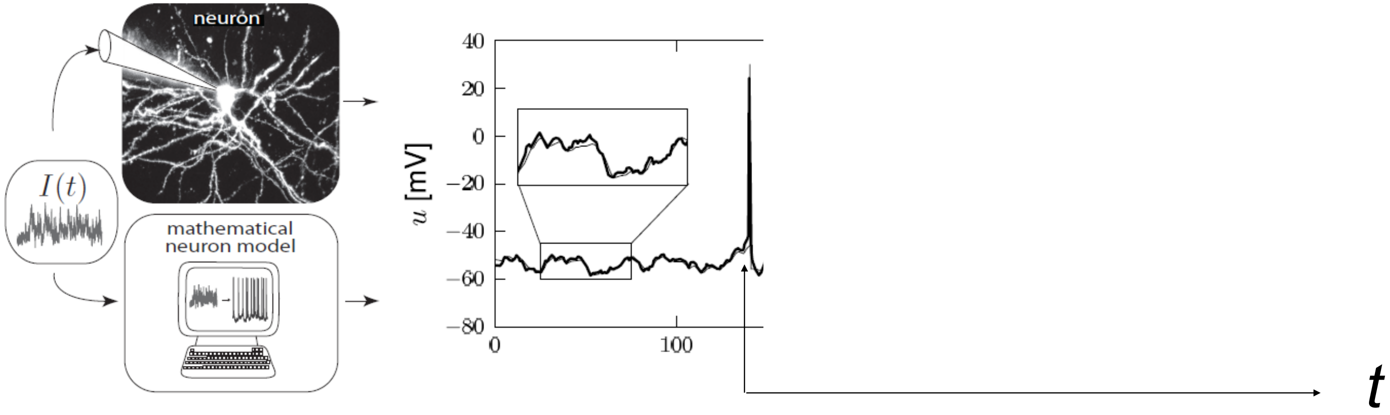
Neuronal Dynamics – 6.2. Interspike Intervals



deterministic part of input

$$I(t) \;\rightarrow\; u(t)$$

noisy part of input/intrinsic noise

→ *escape rate*

Example:
nonlinear integrate-and-fire model

$$\tau \cdot \frac{d}{dt} u = F(u) + RI(t)$$

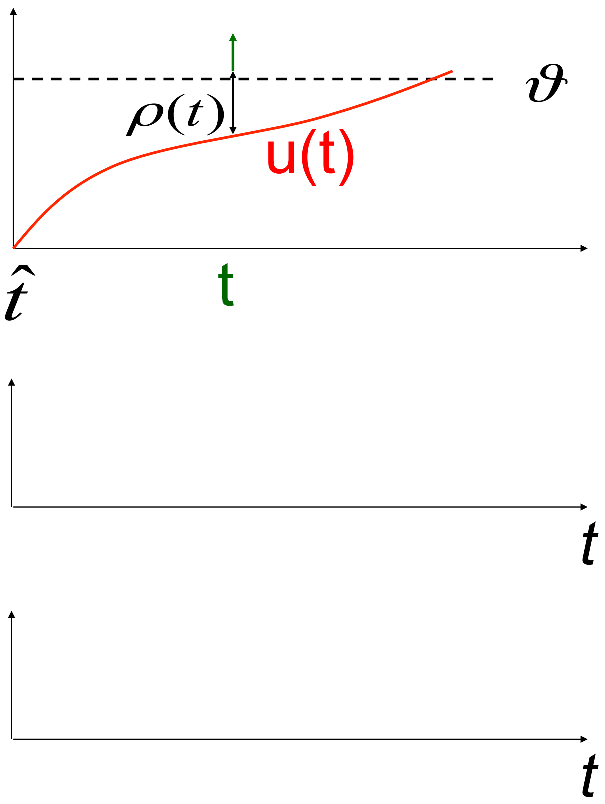
$$if\ spike\ at\ t^f \Rightarrow u\left(t^f + \delta\right) = u_r$$

Example:
exponential stochastic intensity

$$\rho(t) = f(u(t)) = \rho_\vartheta \exp(u(t) - \vartheta)$$

Neuronal Dynamics – 6.2. Interspike Interval distribution

escape process



escape rate

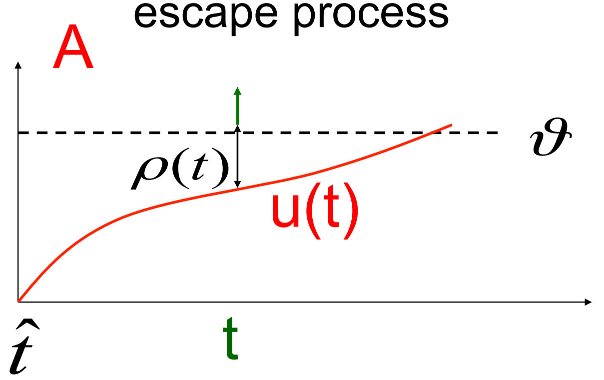
$$\rho(t) = f(u(t) - \vartheta)$$

Survivor function

$$\frac{d}{dt} S_I(t|\hat{t}) = -\rho(t)\, S_I(t|\hat{t})$$

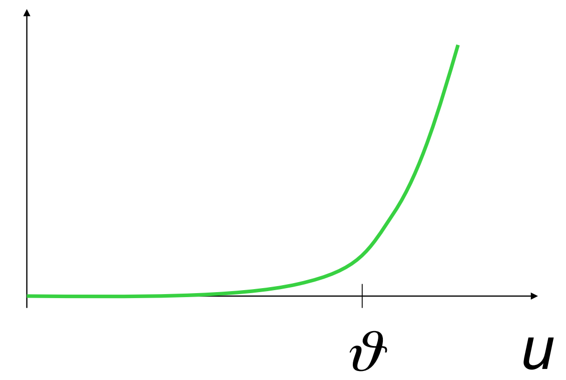
Neuronal Dynamics – 6.2. Interspike Intervals

escape process



escape rate

$$\rho(t) = f(u(t) - \vartheta)$$



Survivor function

Examples now

$$\frac{d}{dt} S_I(t|\hat{t}) = -\rho(t)\, S_I(t|\hat{t})$$

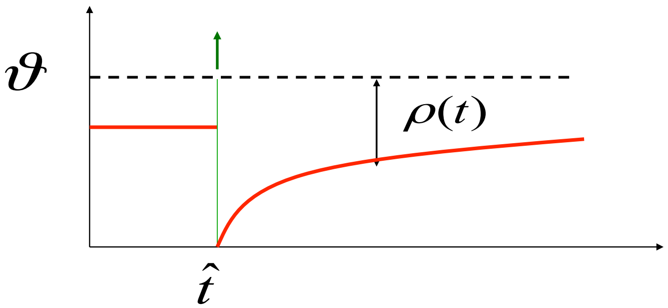
$$S_I(t|\hat{t}) = \underbrace{\exp\left(-\int_{\hat{t}}^{t} \rho(t')dt'\right)}$$

Interval distribution

$$P_I(t|\hat{t}) = \underbrace{\rho(t)}_{\text{escape rate}} \cdot \underbrace{\exp\left(-\int_{\hat{t}}^{t} \rho(t')dt'\right)}_{\text{Survivor function}}$$

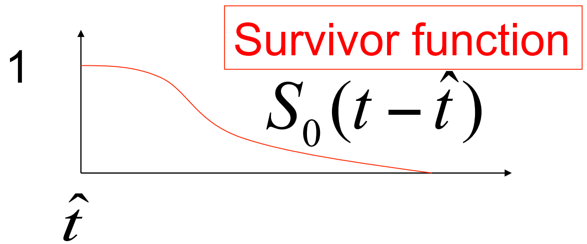
Neuronal Dynamics – 6.2. Renewal theory

Example: I&F with reset, **constant input**

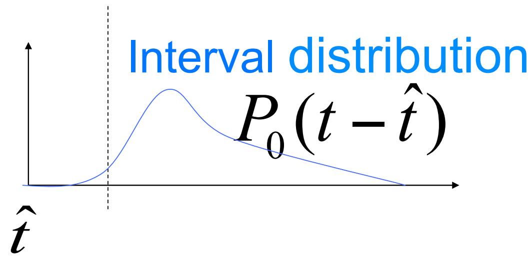


escape rate

$$\rho(t|\hat{t}) = f(u(t|\hat{t})) = \rho_\vartheta \exp(u(t|\hat{t}) - \vartheta)$$



$$\boxed{S(t|\hat{t}) = \exp(-\int_{\hat{t}}^{t} \rho(t'|\hat{t})dt')}$$

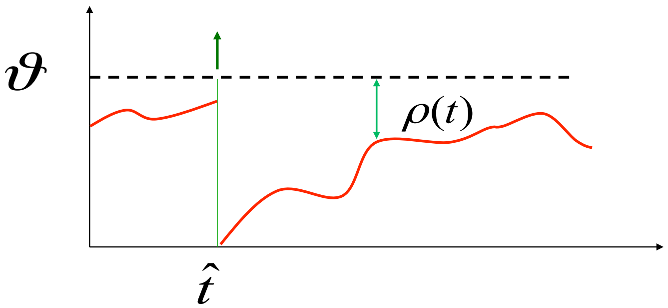


$$P(t|\hat{t}) = \rho(t|\hat{t}) \exp(-\int_{\hat{t}}^{t} \rho(t'|\hat{t})dt')$$

$$= -\frac{d}{dt} S(t|\hat{t})$$

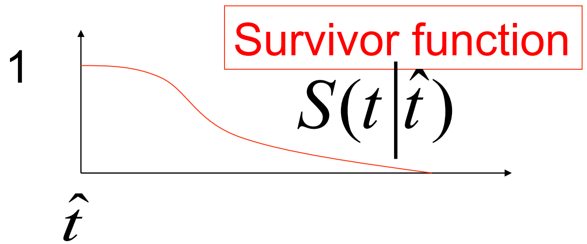
Neuronal Dynamics – 6.2. Time-dependent Renewal theory

Example: I&F with reset, **time-dependent input,**

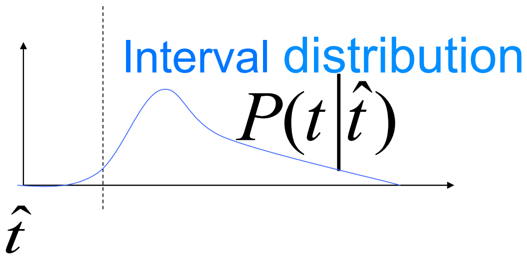


escape rate

$$\rho(t|\hat{t}) = f(u(t|\hat{t})) = \rho_\vartheta \exp(u(t|\hat{t}) - \vartheta)$$



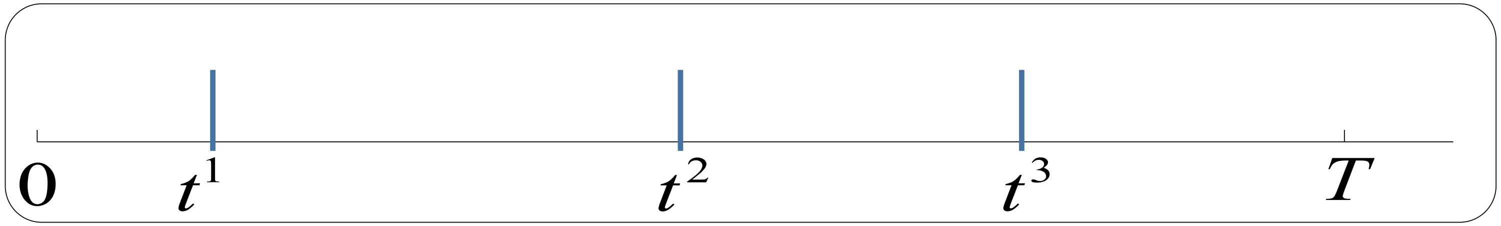
$$S(t|\hat{t}) = \exp\left(-\int_{\hat{t}}^{t} \rho(t'|\hat{t})\,dt'\right)$$



$$P(t|\hat{t}) = \rho(t|\hat{t}) \exp\left(-\int_{\hat{t}}^{t} \rho(t'|\hat{t})\,dt'\right)$$

$$= -\frac{d}{dt} S(t|\hat{t})$$

Neuronal Dynamics – 6.2. Firing probability in discrete time



Probability to survive 1 time step

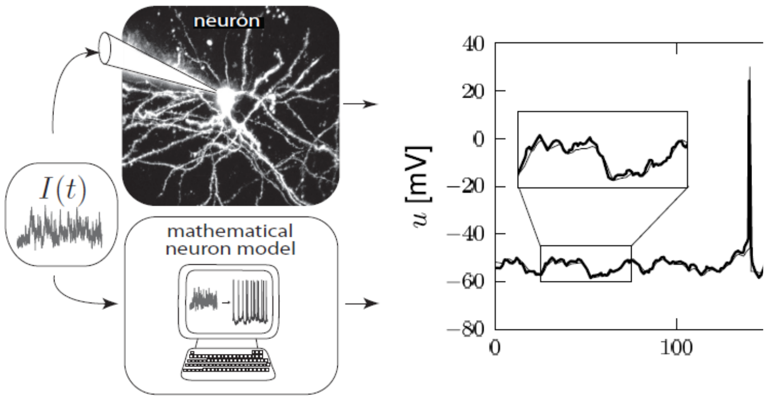
$$\boxed{S(t_{k+1}|t_k) = \exp[-\int_{t_k}^{t_{k+1}} \rho(t')dt']}$$

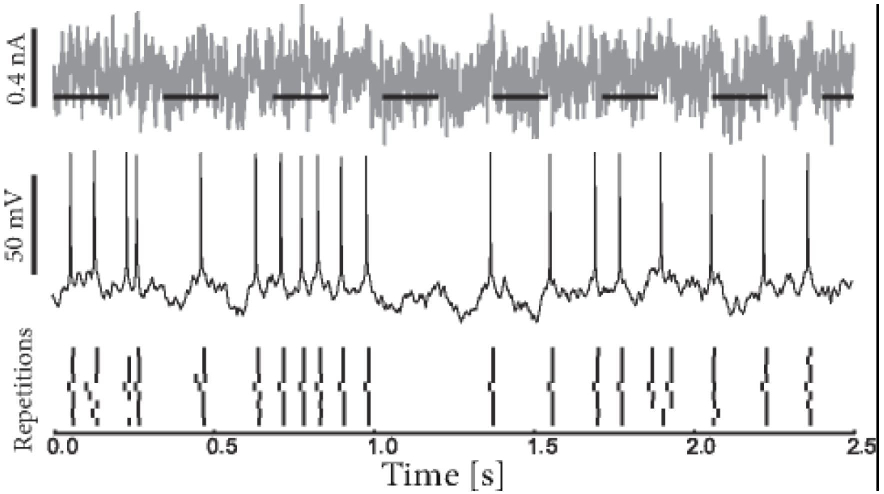
$$S(t_{k+1}\,|\,t_k) = \exp[-\rho(t_k)\Delta] = 1 - P_k^F$$

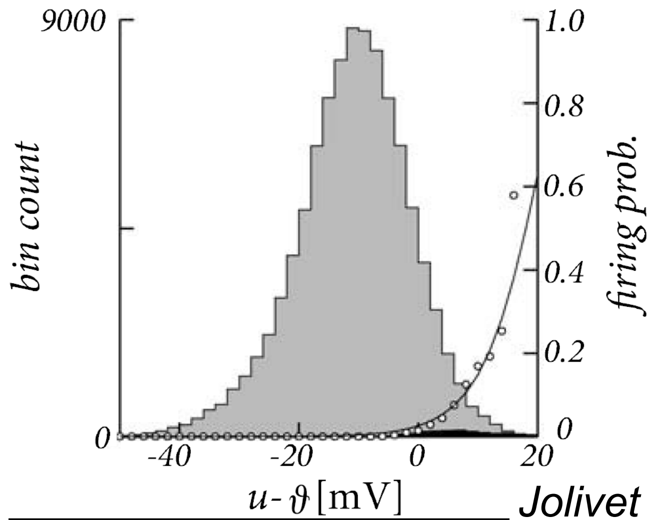
Probability to fire in 1 time step

$$P_k^F =$$

Neuronal Dynamics – 6.2. Escape noise - experiments







Jolivet et al. , J. Comput. Neurosc. 2006

$$P_k^F = 1 - \exp[-\rho(t_k)\Delta]$$

escape rate

$$\rho(t) = \frac{1}{\Delta}\exp\left(\frac{u(t)-\vartheta}{\Delta}\right)$$

Neuronal Dynamics – 6.2. Renewal process, firing probability

Escape noise = stochastic intensity

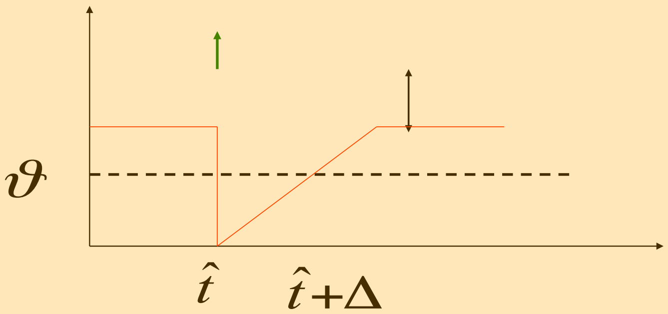
-Renewal theory
- hazard function
- survivor function
- interval distribution

-time-dependent renewal theory
-discrete-time firing probability
-Link to experiments

→ basis for modern methods of neuron model fitting (week 7)

Neuronal Dynamics – Homework assignement 6.1

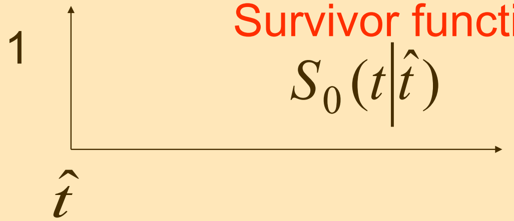
neuron with relative refractoriness, constant input



escape rate

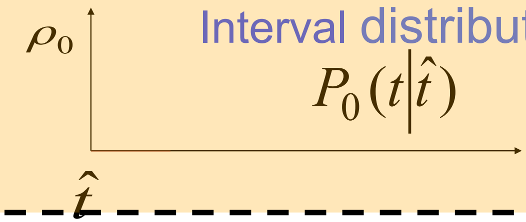
$$\rho(t) = \rho_0 \frac{u}{\vartheta} \quad for \quad u > \vartheta$$

Survivor function $S_0(t|\hat{t})$



$$S_0(t|\hat{t}) = \Bigg\{$$

Interval distribution $P_0(t|\hat{t})$



$$P_0(t|\hat{t}) = \Big\{$$