Week 3 – part 4: Cable equation

Neuronal Dynamics:
Computational Neuroscience of Single Neurons

Week 3 – Adding Detail:
Dendrites and Synapses

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Week 3 – part 4: Cable equation

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

√ 3.1 **Synapses**

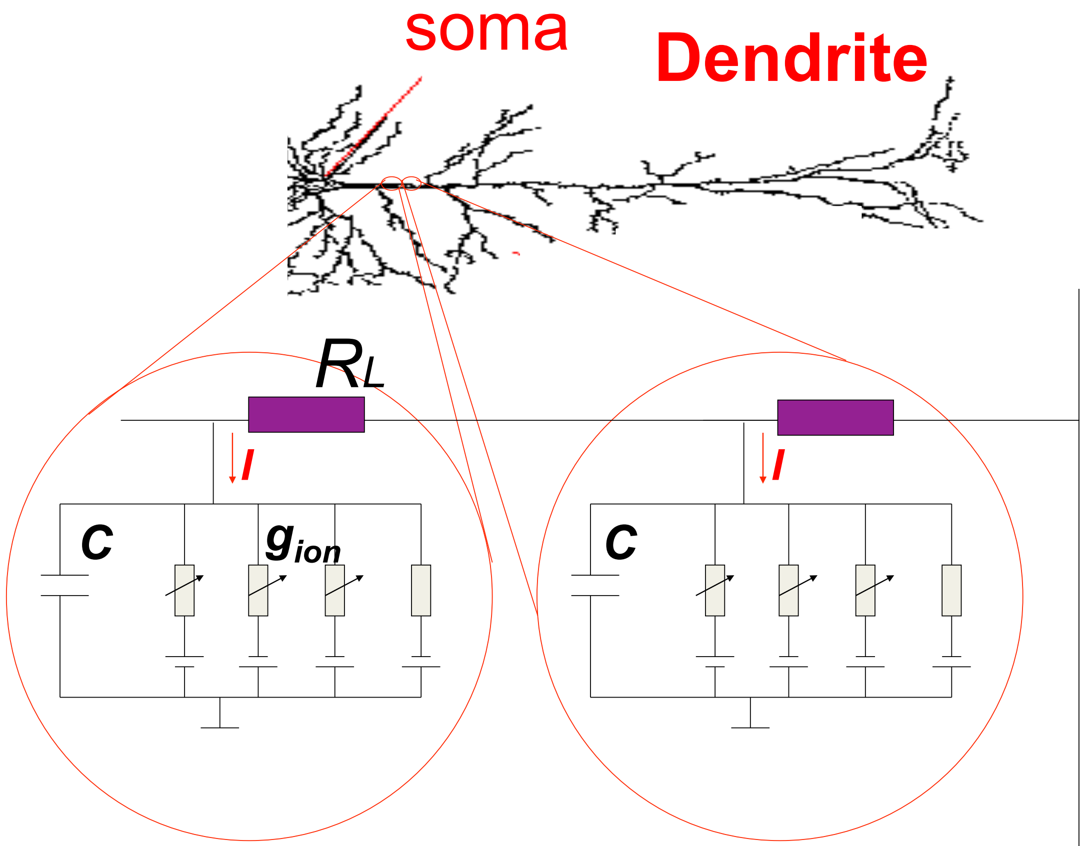
√ 3.2 **Short-term plasticity**

√ 3.3 **Dendrite as a Cable**

$\boxed{\text{3.4 Cable equation}}$

3.5 **Compartmental Models**
- active dendrites

Neuronal Dynamics – 3.4. Cable equation



Neuronal Dynamics – 3.4 Cable equation

$$\frac{d^2}{dx^2}u(t,x) = cr_L \frac{d}{dt}u(t,x) + r_L \sum_{ion} i_{ion}(t,x) - r_L i^{ext}(t,x)$$

$$\boxed{\sum_{ion} i_{ion}(t,x) = leak \quad \text{passive dendrite}}$$

$$\sum_{ion} i_{ion}(t,x) = Ca, Na, \ldots$$ active dendrite

$$\sum_{ion} i_{ion}(t,x) = Na, K, \ldots$$ axon

Neuronal Dynamics – 3.4 Cable equation

Mathematical derivation

$$\frac{d^2}{dx^2}u(t,x) = cr_L \frac{d}{dt}u(t,x) + r_L \sum_{ion} i_{ion}(t,x) - r_L i^{ext}(t,x)$$

$$\sum_{ion} i_{ion}(t,x) = leak$$ passive dendrite

$$\sum_{ion} i_{ion}(t,x) = Ca, Na, \ldots$$ active dendrite

$$\sum_{ion} i_{ion}(t,x) = Na, K, \ldots$$ axon

Neuronal Dynamics – 3.4 Derivation for passive cable

$$\frac{d^2}{dx^2}u(t,x) = cr_L \frac{d}{dt}u(t,x) + r_L \sum_{ion} i_{ion}(t,x) - r_L i^{ext}(t,x)$$

$$\sum_{ion} i_{ion}(t,x) = leak$$ passive dendrite

$$I_{ion} = i_{ion}\, dx$$

$$I^{ext} = i^{ext}\, dx$$

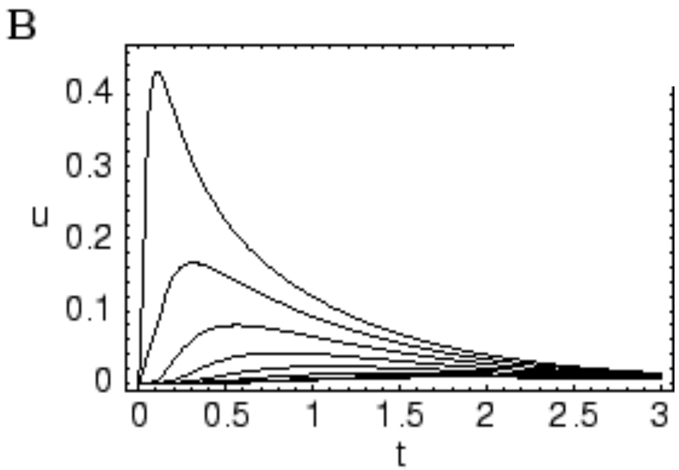
$$\lambda^2 \frac{d^2}{dx^2}u(t,x) = \tau_m \frac{d}{dt}u(t,x) + u - r_m i^{ext}(t,x)$$

$$\sum_{ion} i_{ion}(t,x) = \frac{u}{r_m}$$

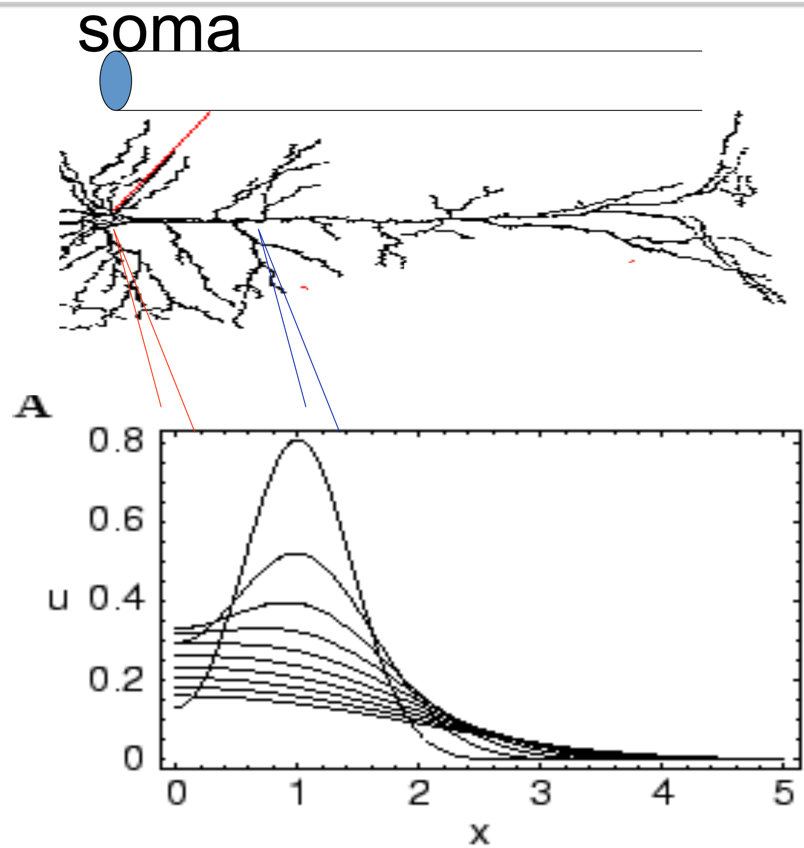
Neuronal Dynamics – 3.4 dendritic stimulation

passive dendrite/passive cable

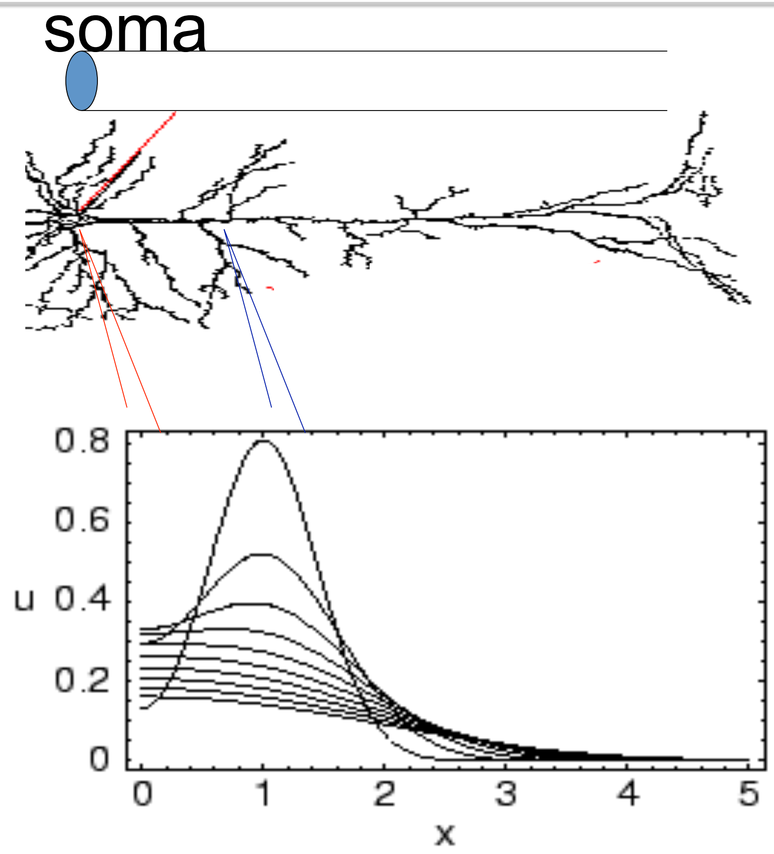
$$\lambda^2 \frac{d^2}{dx^2} u(t,x) = \tau_m \frac{d}{dt} u(t,x) + u - r_m i^{ext}(t,x)$$



Stimulate dendrite, measure at soma



Neuronal Dynamics – 3.4 dendritic stimulation



Neuronal Dynamics – Quiz 3.4

Multiple answers possible!

The space constant of a passive cable is

[ ] $\lambda = \frac{r_m}{r_L}$

[ ] $\lambda = \frac{r_L}{r_m}$

[ ] $\lambda = \sqrt{\frac{r_L}{r_m}}$

[ ] $\lambda = \sqrt{\frac{r_m}{r_L}}$

Dendritic current injection.

If a short current pulse is injected into the dendrite

[ ] the voltage at the injection site is maximal immediately after the end of the injection

[ ] the voltage at the dendritic injection site is maximal a few milliseconds after the end of the injection

[ ] the voltage at the soma is maximal immediately after the end of the injection.

[ ] the voltage at the soma is maximal a few milliseconds after the end of the injection

It follows from the cable equation that

[ ] the shape of an EPSP depends on the dendritic location of the synapse.

[ ] the shape of an EPSP depends only on the synaptic time constant, but not on dendritic location.

Neuronal Dynamics – Homework 3.1

Consider

$$(*)\quad u(t,x) = \frac{1}{\sqrt{4\pi t}} \exp[-t - \frac{(x-x_0)^2}{4t}] \quad for\ t>0$$

$$u(t,x) = 0 \quad for\ t<0$$

(i) Take the second derivative of (*) with respect to *x*. The result is

$$\frac{d^2}{dx^2} u(t,x) = ...$$

(ii) Take the derivative of (*) with respect to *t.* The result is

$$\frac{d}{dt} u(t,x) = ...$$

(iii) Therefore the equation is a solution to

$$\lambda^2 \frac{d^2}{dx^2} u(t,x) = \tau_m \frac{d}{dt} u(t,x) + u - r_m i^{ext}(t,x)$$

with $\tau_m = ...and\ \ \lambda = ...$

(iv) The input current is [ ] $i^{ext}(t,x) = \delta(t)\delta(x-x_0)$

[ ] $i^{ext}(t,x) = i_0 \quad for\ \ t>0$

Neuronal Dynamics – Homework 3.2

Consider the two equations

(1) $$\lambda^2 \frac{d^2}{dx^2} u(t,x) = \tau_m \frac{d}{dt} u(t,x) + u - r_m i^{ext}(t,x)$$

(2) $$\frac{d^2}{d\tilde{x}^2} u(t',\tilde{x}) = \frac{d}{dt'} u(t',\tilde{x}) + u - i^{ext}(t',\tilde{x})$$

The two equations are equivalent under the transform

$$\tilde{x} = cx \quad and \quad t' = at$$

with constants c= ….. and a = …..