Week 4 – part 5: Nonlinear Integrate-and-Fire Model

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Neuronal Dynamics:
Computational Neuroscience of Single Neurons

Week 4 – Reducing detail:
Two-dimensional neuron models

Wulfram Gerstner

EPFL, Lausanne, Switzerland

√4.1 **From Hodgkin-Huxley to 2D**

√4.2 **Phase Plane Analysis**

√4.3 **Analysis of a 2D Neuron Model**

√4.4 **Type I and II Neuron Models**
- where is the firing threshold?
- MathDetour 4: separation of time scales

4.5. **Nonlinear Integrate-and-fire**
- from two to one dimension

Week 4 – part 5: Nonlinear Integrate-and-Fire Model

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√4.1 **From Hodgkin-Huxley to 2D**

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Neuronal Dynamics – 4.5. Further reduction to 1 dimension

2-dimensional equation

stimulus

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

slow!

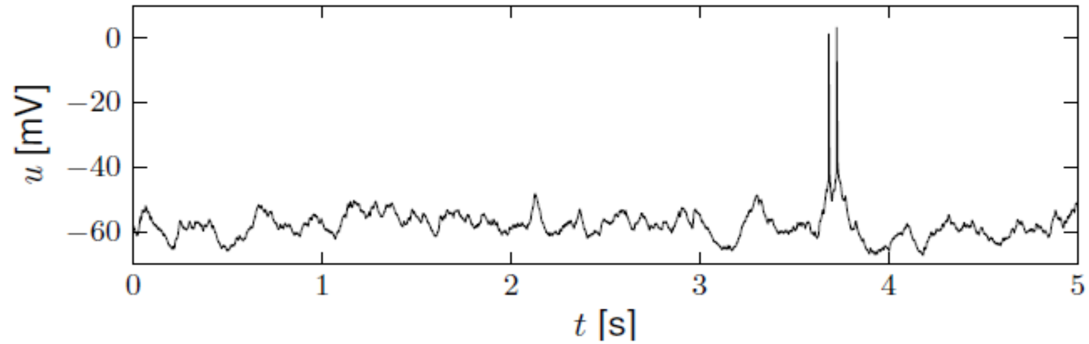
Separation of time scales

-*w* is nearly constant
(most of the time)

Neuronal Dynamics – 4.5 sparse activity in vivo

Spontaneous activity *in vivo*

awake mouse, cortex, freely whisking,



-spikes are rare events

Crochet et al., 2011

-membrane potential fluctuates around ‘rest’

Aims of Modeling: - predict spike initation times
- predict subthreshold voltage

Neuronal Dynamics – 4.5. Further reduction to 1 dimension

stimulus

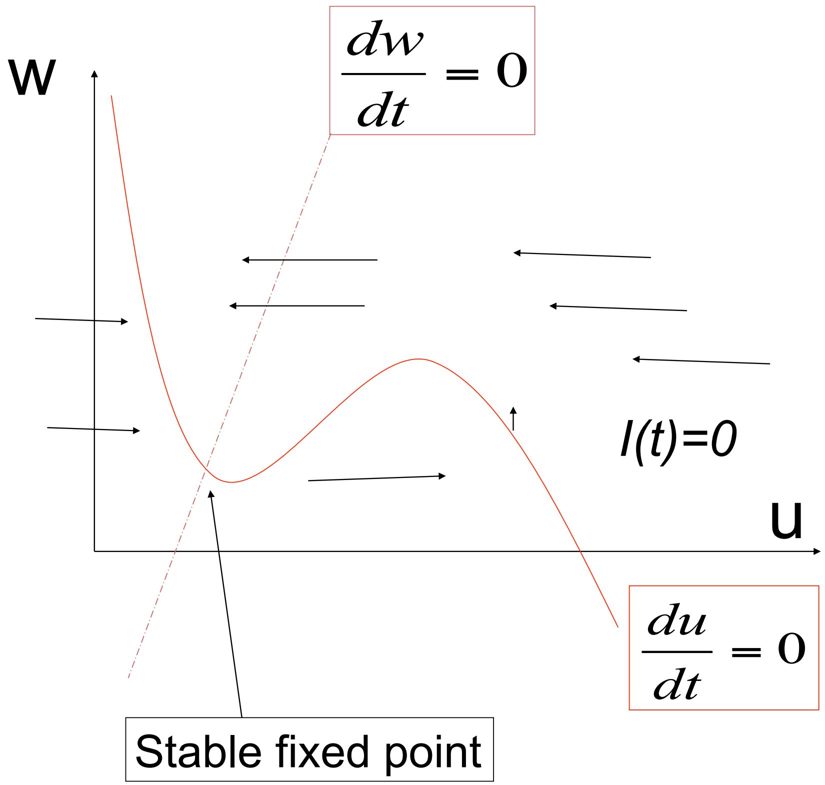
$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

Separation of time scales

$$\tau_w >> \tau_u$$

→ Flux nearly horizontal



Neuronal Dynamics – 4.5. Further reduction to 1 dimension

Hodgkin-Huxley reduced to 2dim

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

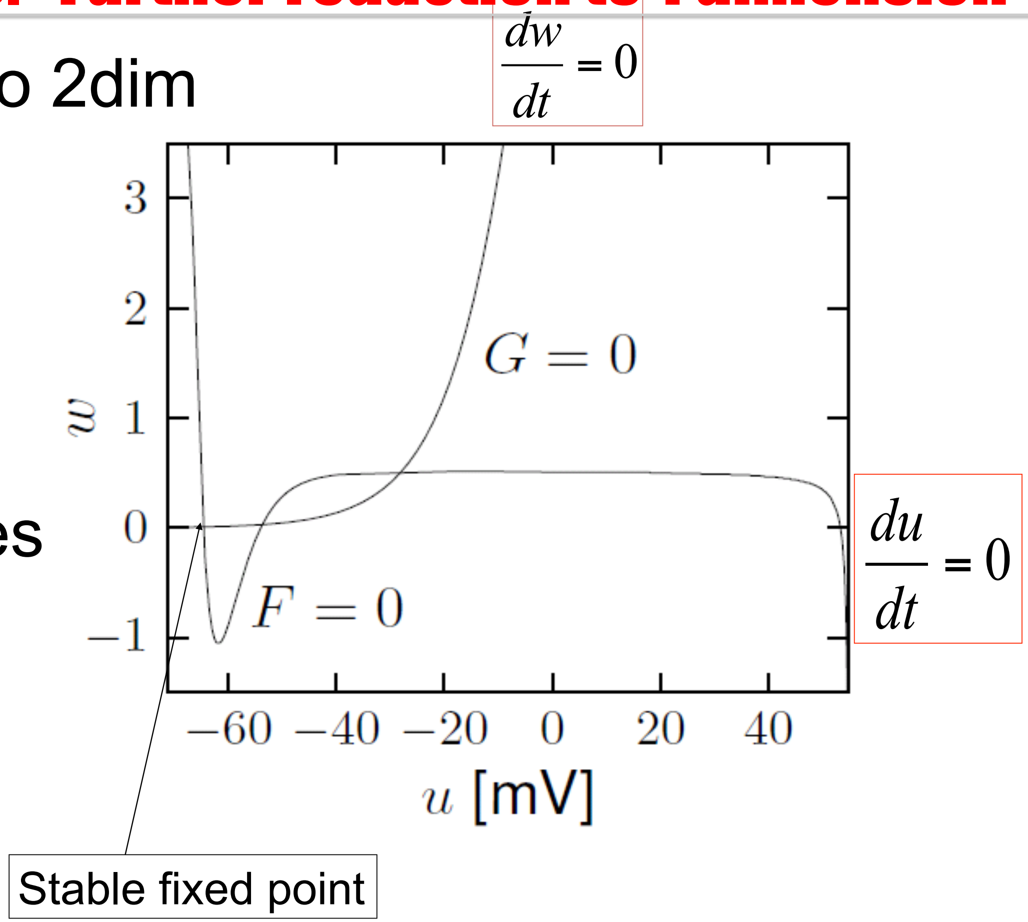
$$\tau_w \frac{dw}{dt} = G(u,w)$$

Separation of time scales

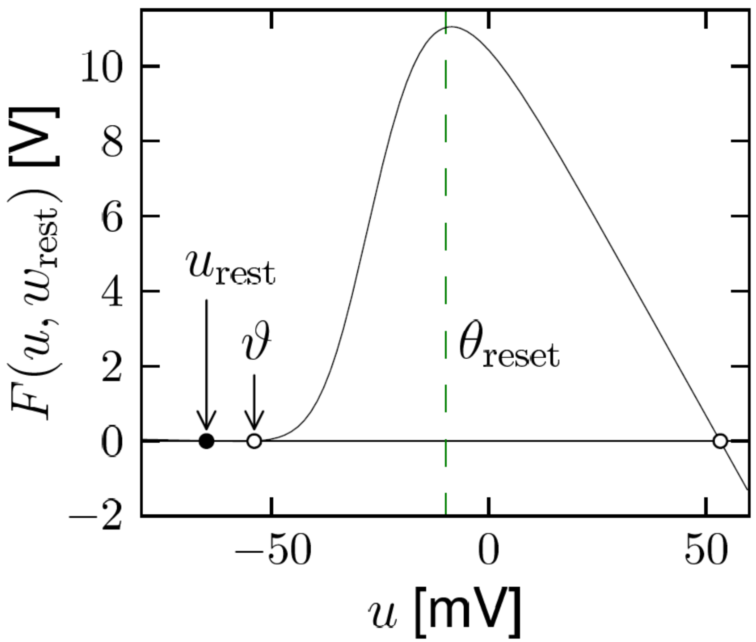
$$\tau_w >> \tau_u$$

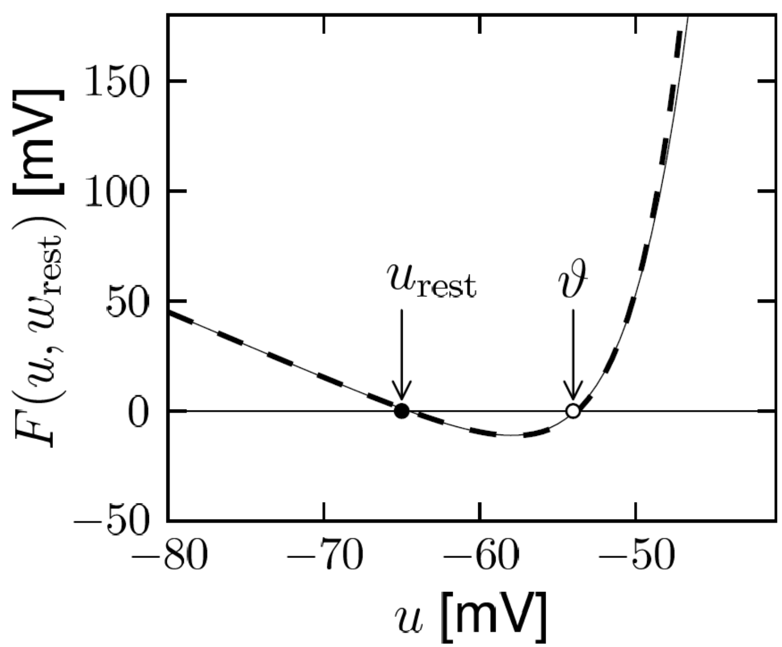
$$\tau_w \frac{dw}{dt} \approx 0 \rightarrow \quad w \approx w_{rest}$$

$$\tau \frac{du}{dt} = F(u, w_{rest}) + RI(t)$$



Neuronal Dynamics – 4.5. Nonlinear Integrate-and-Fire Model





$$\tau \frac{du}{dt} = F(u, w_{rest}) + RI(t) = f(u) + RI(t)$$

→ Nonlinear I&F (see week 1!)

Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

Neuronal Dynamics – 4.5. Nonlinear Integrate-and-Fire Model

Exponential integrate-and-fire model (EIF)

$$\boxed{f(u) = -(u - u_{rest}) + \Delta \exp(\tfrac{u-\vartheta}{\Delta})}$$

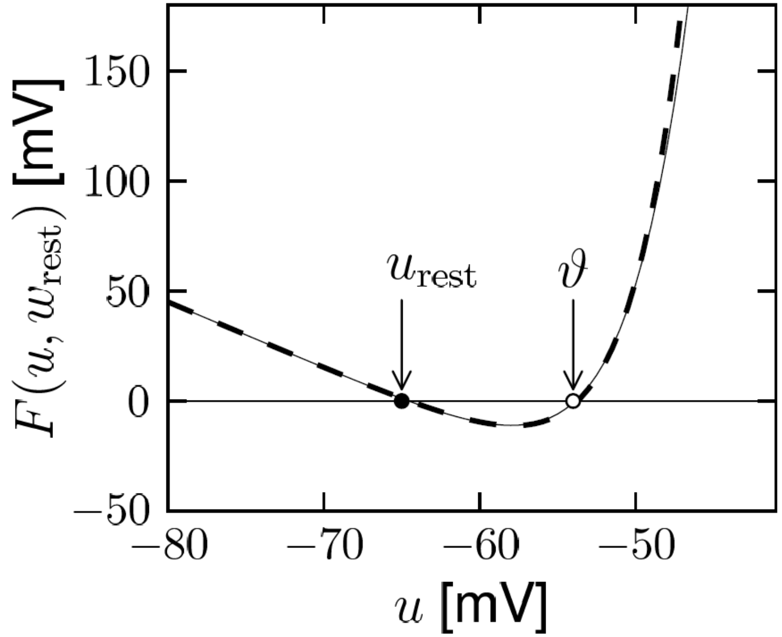


Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

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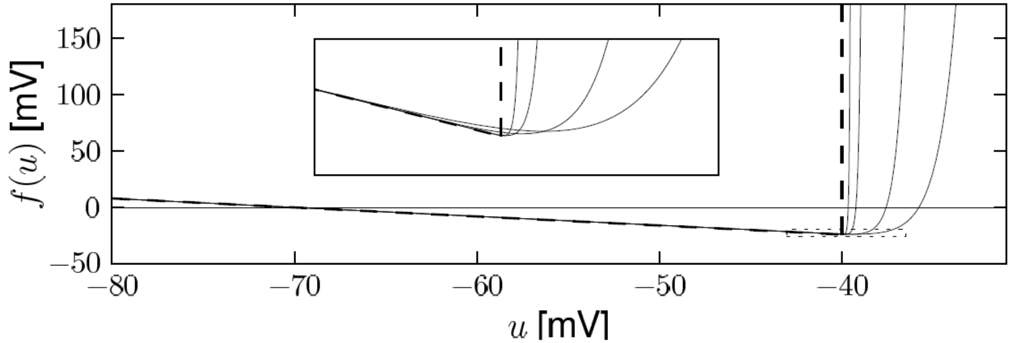
→ Nonlinear I&F (see week 1!)

Exponential integrate-and-fire model (EIF)

$$\boxed{f(u) = -(u - u_{rest}) + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right)}$$

linear



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Neuronal Dynamics – 4.5 sparse activity in vivo

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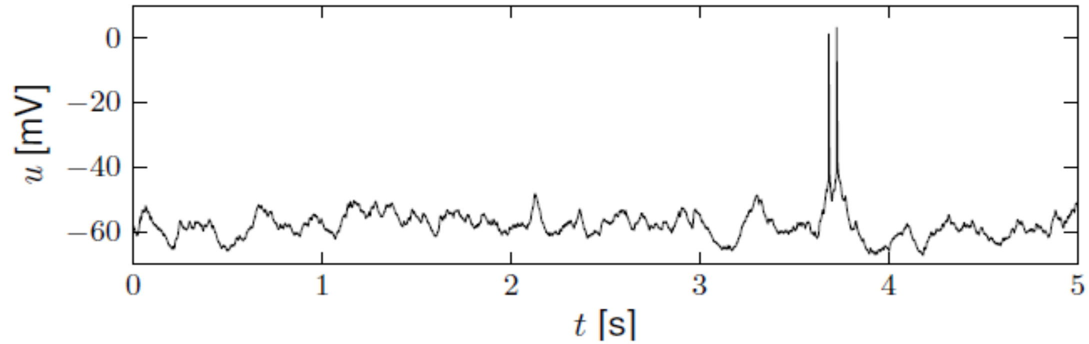
awake mouse, cortex, freely whisking,



-spikes are rare events

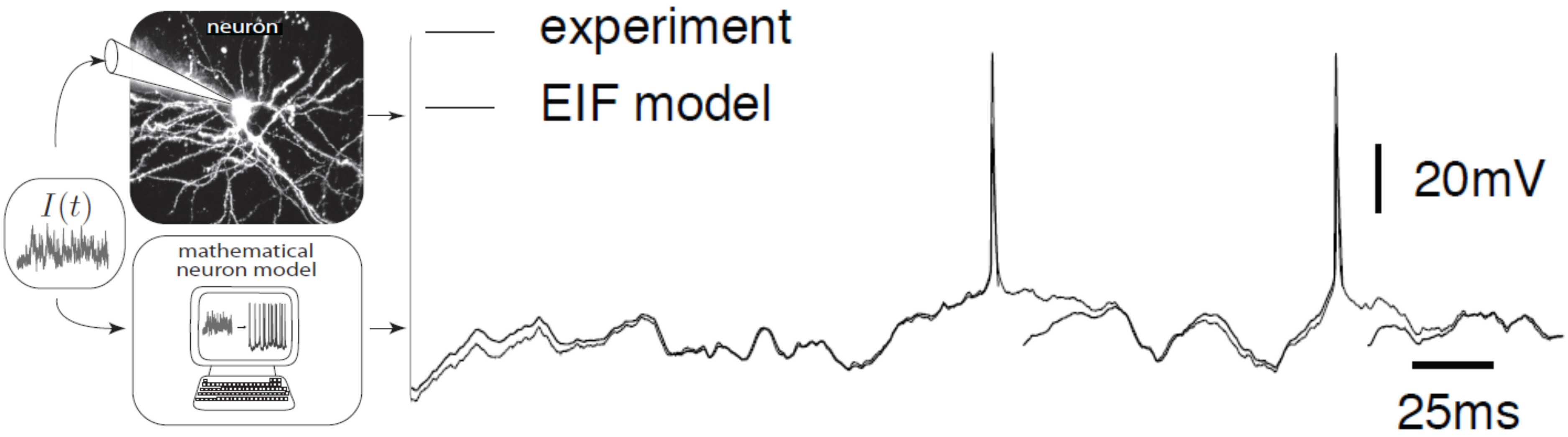
Crochet et al., 2011

-membrane potential fluctuates around 'rest'

Aims of Modeling: - predict spike initation times

- predict subthreshold voltage

Neuronal Dynamics – 4.5.How good are integrate-and-fire models?



Badel et al., 2008

Aims: - predict spike initation times
- predict subthreshold voltage

Add adaptation and refractoriness (week 7)

Neuronal Dynamics – 4.5. Exponential Integrate-and-Fire Model

Direct derivation from Hodgkin-Huxley

$$C\frac{du}{dt} = -g_{Na}\, m^3\, h\,(u - E_{Na}) - g_K n^4\,(u - E_K) - g_l(u - E_l) + I(t)$$

$$C\frac{du}{dt} = -g_{Na}[m_0(u)]^3\, h_{rest}(u - E_{Na}) - g_K[n_{rest}]^4(u - E_K) - g_l(u - E_l) + I(t)$$

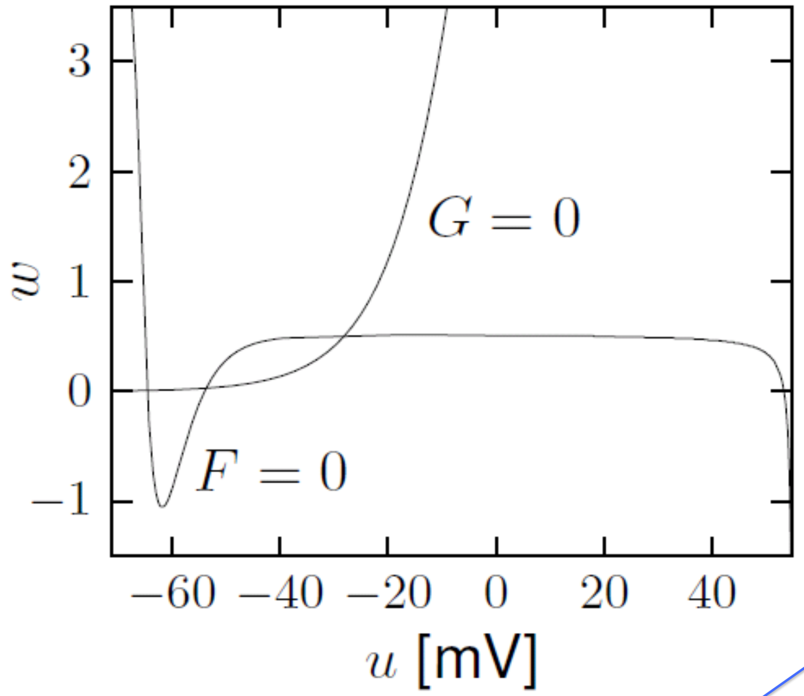
Fourcaud-Trocme et al, J. Neurosci. 2003

$$\boxed{f(u) = -(u - u_{rest}) + \Delta\exp(\tfrac{u-\vartheta}{\Delta})}$$

$$\tau\frac{du}{dt} = F(u, h_{rest}, n_{rest}) + RI(t) = f(u) + RI(t)$$

gives expon. I&F

Neuronal Dynamics – 4.5. Nonlinear Integrate-and-Fire Model



2-dimensional equation

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

Relevant during spike and downswing of AP

Separation of time scales

-w is constant (if not firing)

$$\tau \frac{du}{dt} = f(u) + RI(t)$$

threshold+reset for firing

2-dimensional equation

$$\tau \frac{du}{dt} = F(u, w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u, w)$$

Separation of time scales

-*w* is constant (if not firing)

$$\tau \frac{du}{dt} = f(u) + RI(t)$$

Linear plus exponential

Neuronal Dynamics – Quiz 4.7.

A. Exponential integrate-and-fire model.

The model can be derived

[ ] from a 2-dimensional model, assuming that the auxiliary variable w is constant.
[ ] from the HH model, assuming that the gating variables h and n are constant.
[ ] from the HH model, assuming that the gating variables m is constant.
[ ] from the HH model, assuming that the gating variables m is instantaneous.

B. Reset.

[ ] In a 2-dimensional model, the auxiliary variable w is necessary to implement a reset of the voltage after a spike
[ ] In a nonlinear integrate-and-fire model, the auxiliary variable w is necessary to implement a reset of the voltage after a spike
[ ] In a nonlinear integrate-and-fire model, a reset of the voltage after a spike is implemented algorithmically/explicitly

Neuronal Dynamics – 4.4 Literature

Reading: W. Gerstner, W.M. Kistler, R. Naud and L. Paninski,
Neuronal Dynamics: from single neurons to networks and models of cognition. Chapter 4*: Introduction*. Cambridge Univ. Press, 2014
OR W. Gerstner and W.M. Kistler, *Spiking Neuron Models*, Ch.3. Cambridge 2002
OR J. Rinzel and G.B. Ermentrout, (1989). Analysis of neuronal excitability and oscillations. In Koch, C. Segev, I., editors, *Methods in neuronal modeling*. MIT Press, Cambridge, MA.

Selected references.

-Ermentrout, G. B. (1996). *Type I membranes, phase resetting curves, and synchrony*. Neural Computation, 8(5):979-1001.
-Fourcaud-Trocme, N., Hansel, D., van Vreeswijk, C., and Brunel, N. (2003). *How spike generation mechanisms determine the neuronal response to fluctuating input.* J. Neuroscience, 23:11628-11640.
-Badel, L., Lefort, S., Berger, T., Petersen, C., Gerstner, W., and Richardson, M. (2008). Biological Cybernetics, 99(4-5):361-370.
- E.M. Izhikevich, Dynamical Systems in Neuroscience, MIT Press (2007)