

## Week 1 – part 2: The Passive Membrane



# Neuronal Dynamics: Computational Neuroscience of Single Neurons

Week 1 – neurons and mathematics:  
a first simple neuron model

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### ✓ 1.1 Neurons and Synapses:

Overview

### 1.2 The Passive Membrane

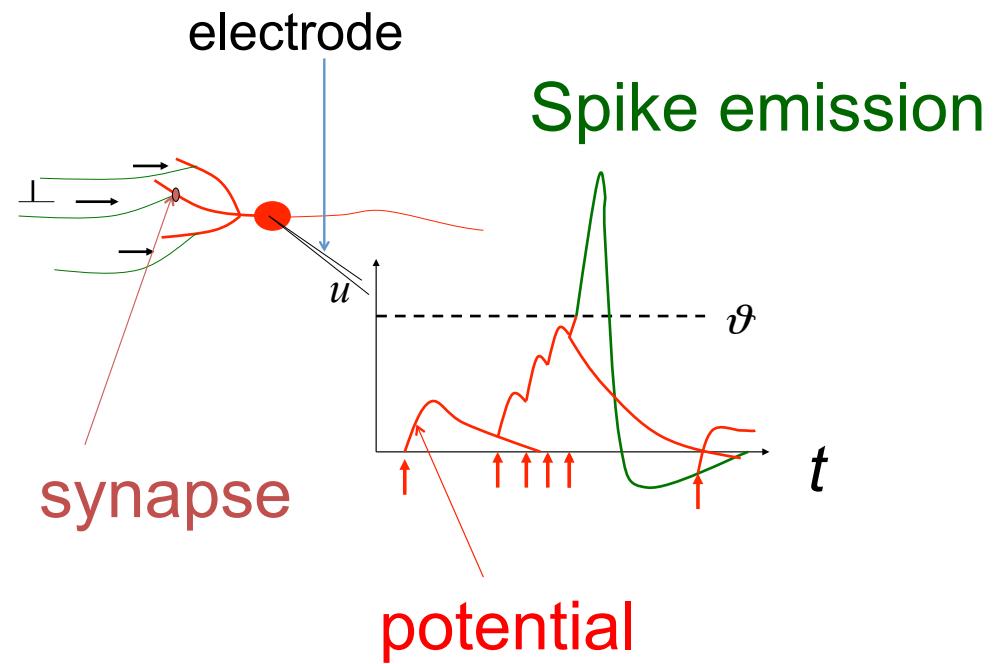
- Linear circuit
- Dirac delta-function

### 1.3 Leaky Integrate-and-Fire Model

### 1.4 Generalized Integrate-and-Fire Model

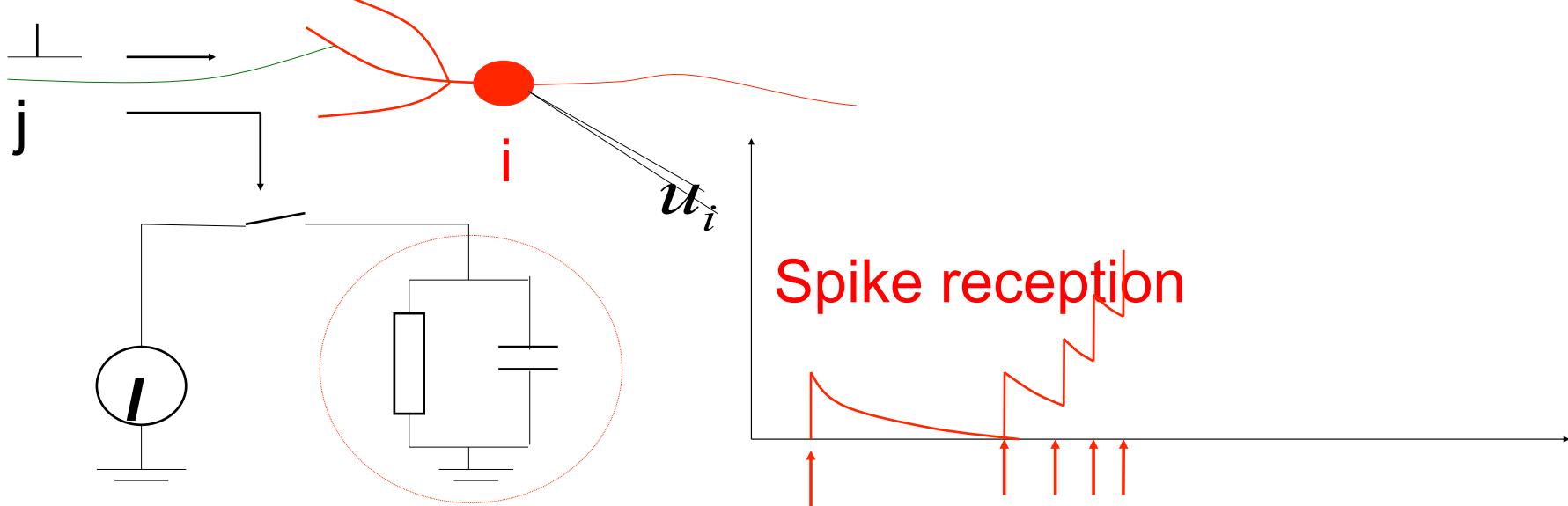
### 1.5. Quality of Integrate-and-Fire Models

# Neuronal Dynamics – 1.2. The passive membrane



Integrate-and-fire model

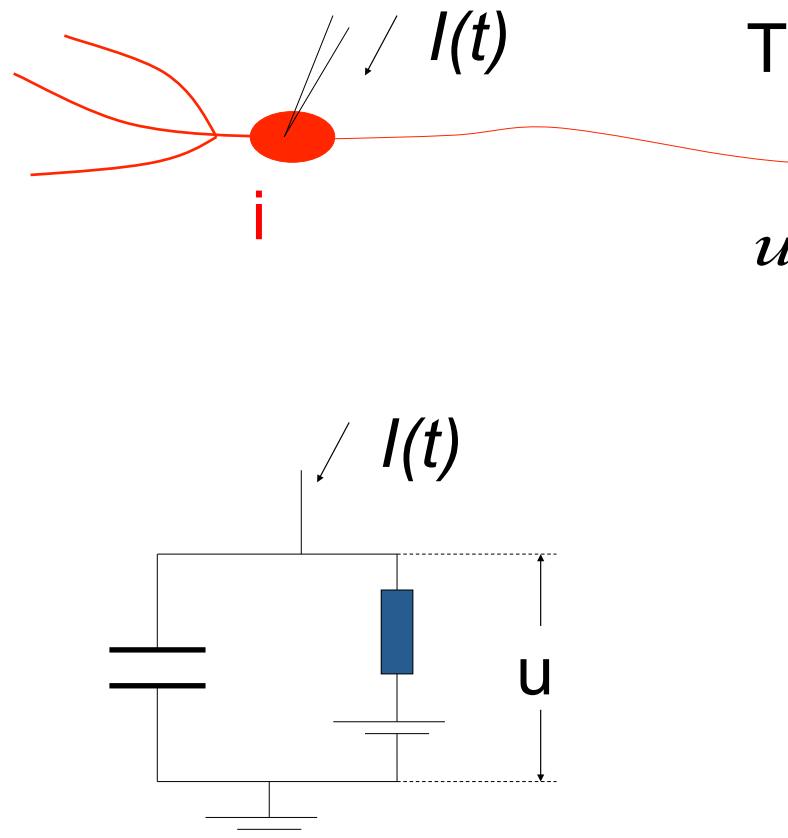
## Neuronal Dynamics – 1.2. The passive membrane



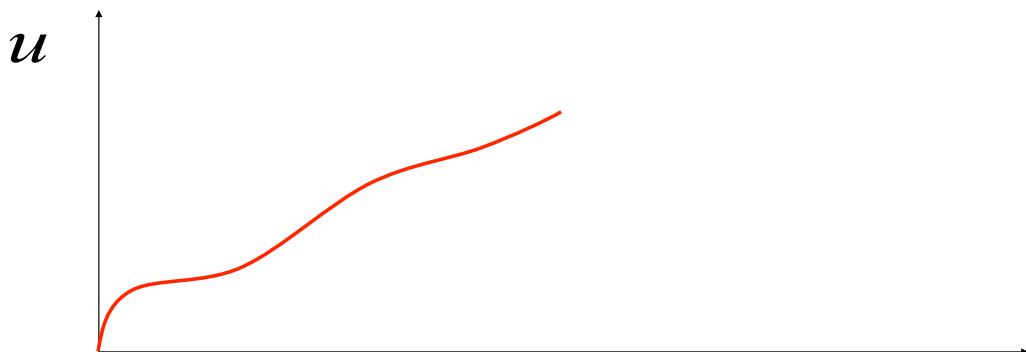
### Subthreshold regime

- linear
- passive membrane
- RC circuit

# Neuronal Dynamics – 1.2. The passive membrane

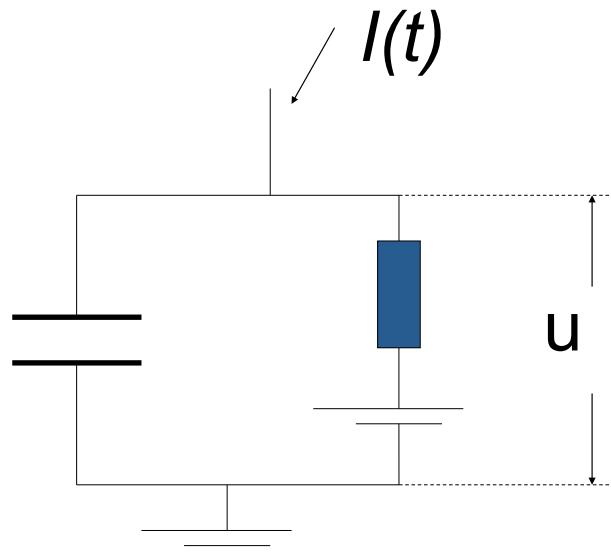


Time-dependent input

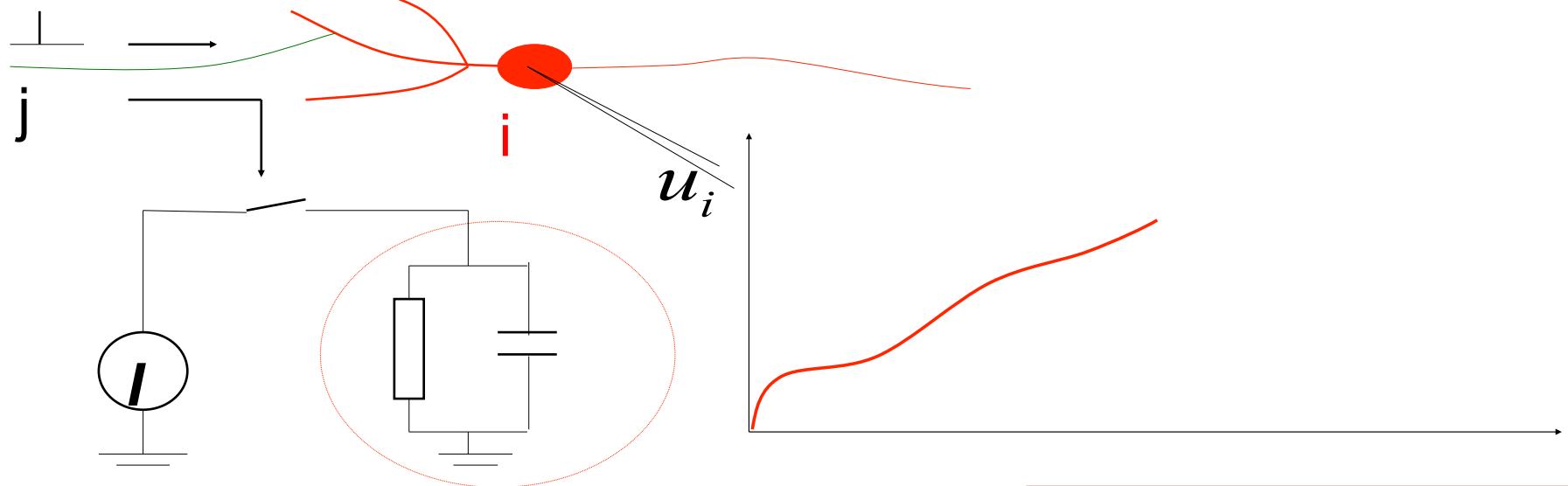


*Math development:  
Derive equation*

# Passive Membrane Model



# Passive Membrane Model



$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$
$$\tau \cdot \frac{d}{dt} V = -V + RI(t); \quad V = (u - u_{rest})$$

*Math Development:  
Voltage rescaling*

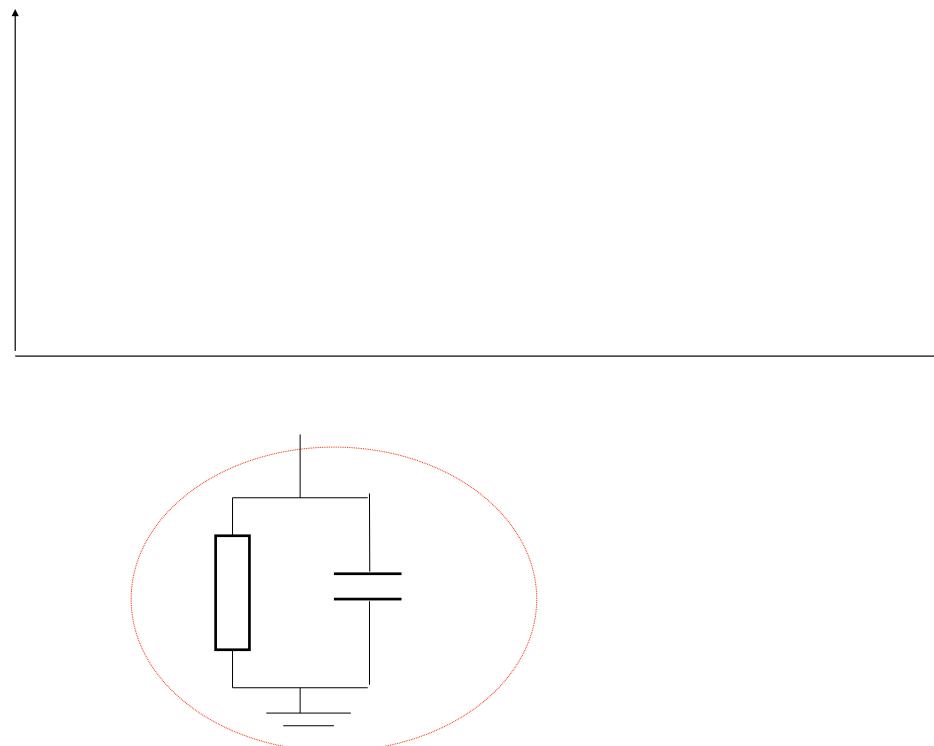
## Passive Membrane Model

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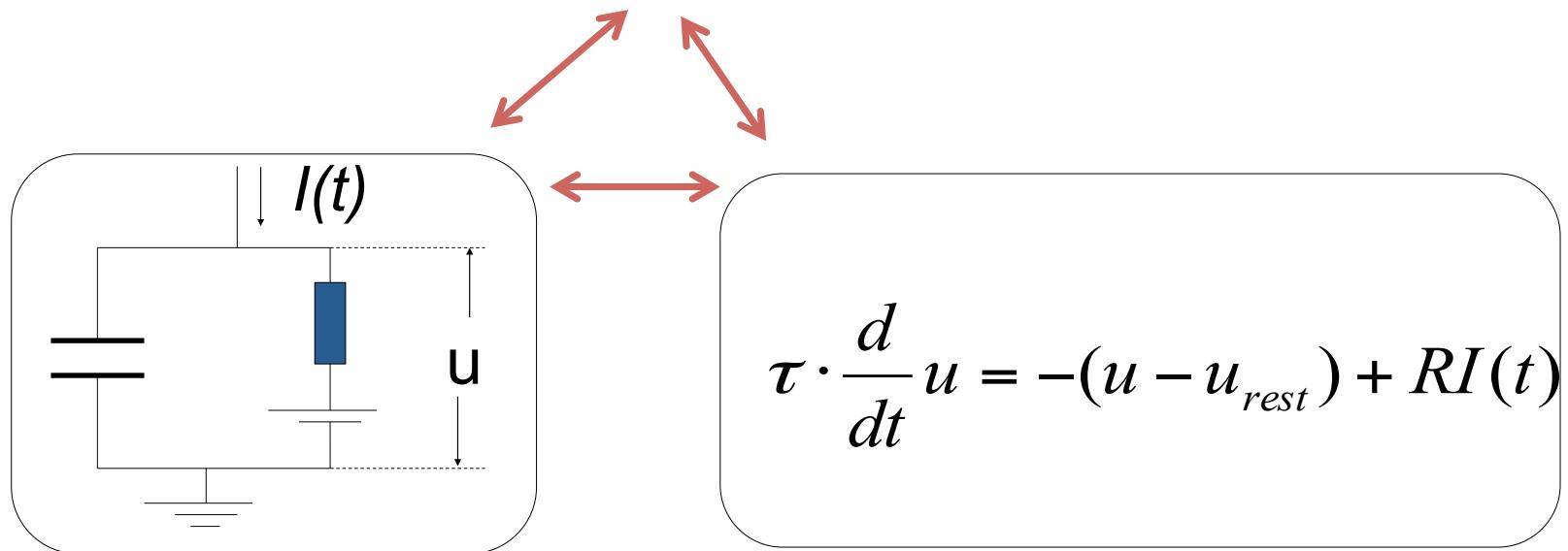
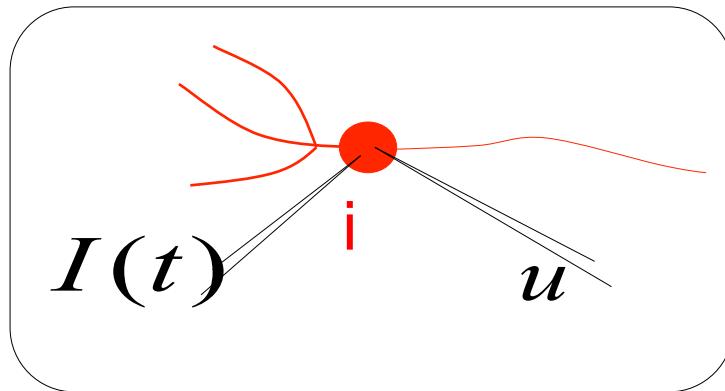
# Passive Membrane Model/Linear differential equation

$$\tau \cdot \frac{d}{dt} V = -V + RI(t);$$

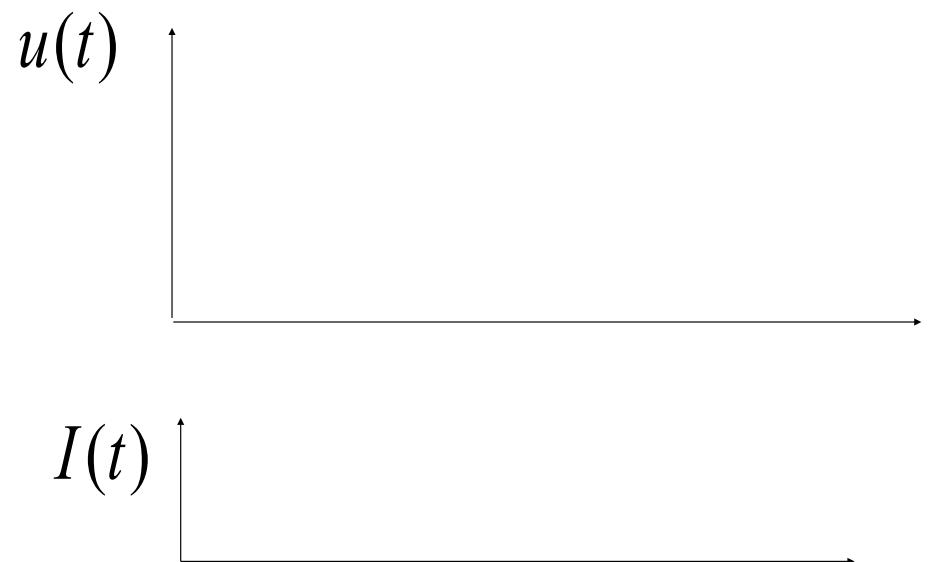
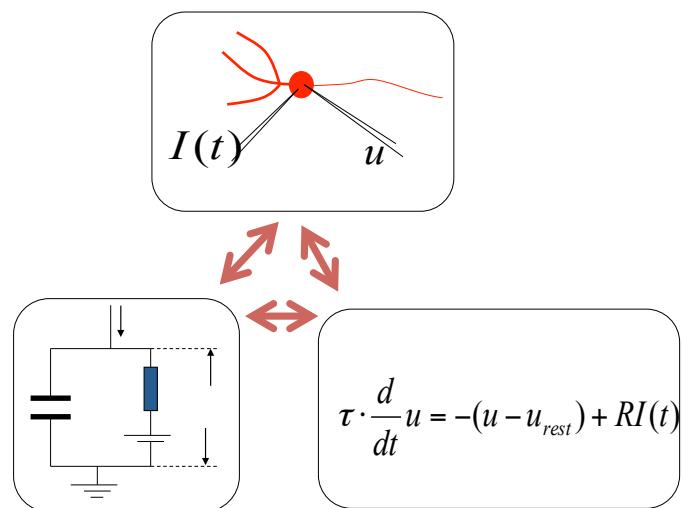


Free solution:  
exponential decay

# Triangle: neuron – electricity - math



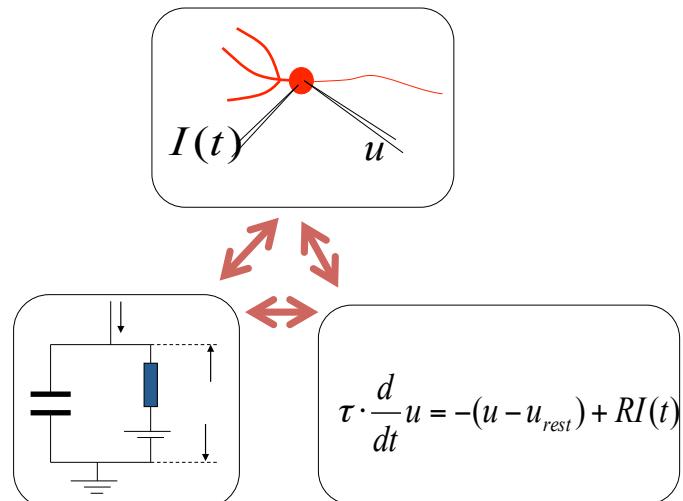
# Pulse input – charge – delta-function



$$I(t) = q \cdot \delta(t - t_0) \quad \text{Pulse current input}$$

# Dirac delta-function

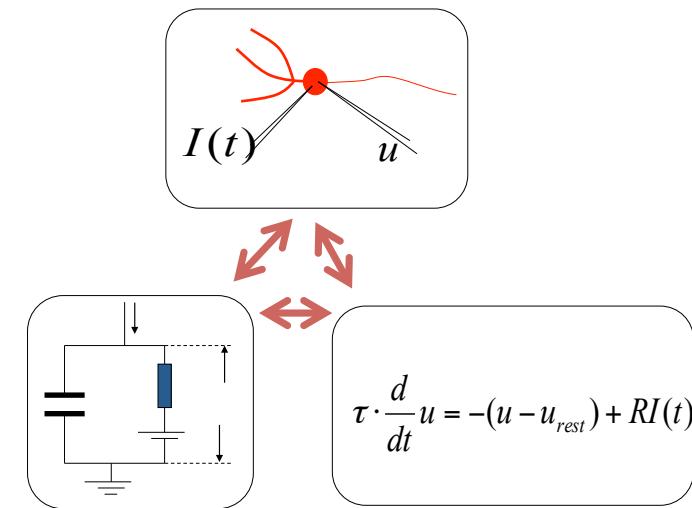
$$I(t) = q \cdot \delta(t - t_0)$$



$$1 = \int_{t_0-a}^{t_0+a} \delta(t - t_0) dt$$

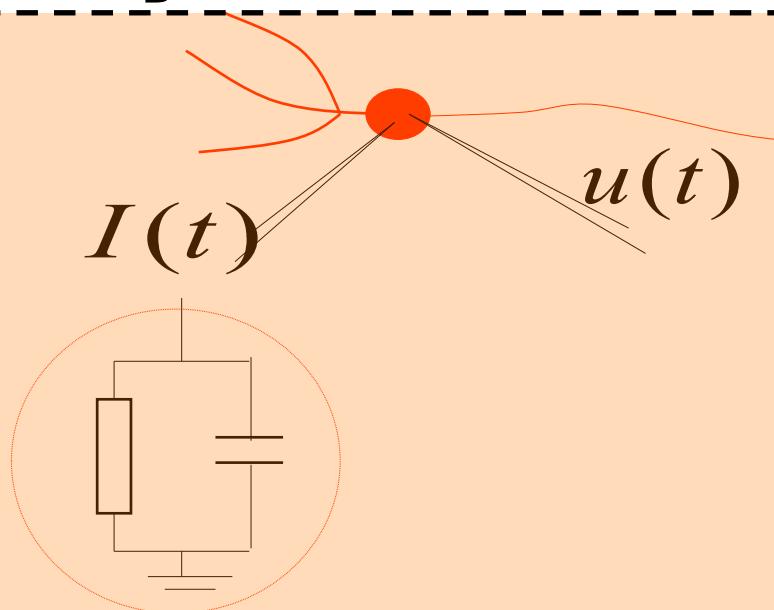
$$f(t_0) = \int_{t_0-a}^{t_0+a} f(t) \delta(t - t_0) dt$$

# Passive membrane, linear differential equation



*Spend 10-15 minutes on  
Homework 1.1 now!  
If you have difficulties,  
watch lecture 1.2 detour.*

# Neuronal Dynamics – Exercises 1.2 = Homework 1.1



$I_1(t)$

Step current input:

$I_2(t)$

Pulse current input:

$I_3(t)$

arbitrary current input:

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

$$\tau \cdot \frac{d}{dt} V = -V + RI(t); \quad V = (u - u_{rest})$$

**Calculate the voltage,  
for the  
3 input currents**