Week 4 – MathDetour 2: Exploiting similarities

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Neuronal Dynamics:

Computational Neuroscience of Single Neurons

Week 4 – Reducing detail: Two-dimensional neuron models

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4.1 **From Hodgkin-Huxley to 2D**
- Overview: From 4 to 2 dimensions
- MathDetour 1: Separation of time scales
- MathDetour 2: Exploiting similarities

4.2 **Phase Plane Analysis**
- role of nullclines
- MathDetour 3: Stability of fixed points

4.3 **Analysis of a 2D Neuron Model**

4.4 **Type I and II Neuron Models**
- where is the firing threshold?
- separation of time scales

4.5. **Nonlinear Integrate-and-fire**
- from two to one dimension

Reduction of Hodgkin-Huxley Model to 2 Dimension

-step 1:
separation of time scales
(→ 4.1 and 4-Detour1)

-step 2:
exploit similarities/correlations

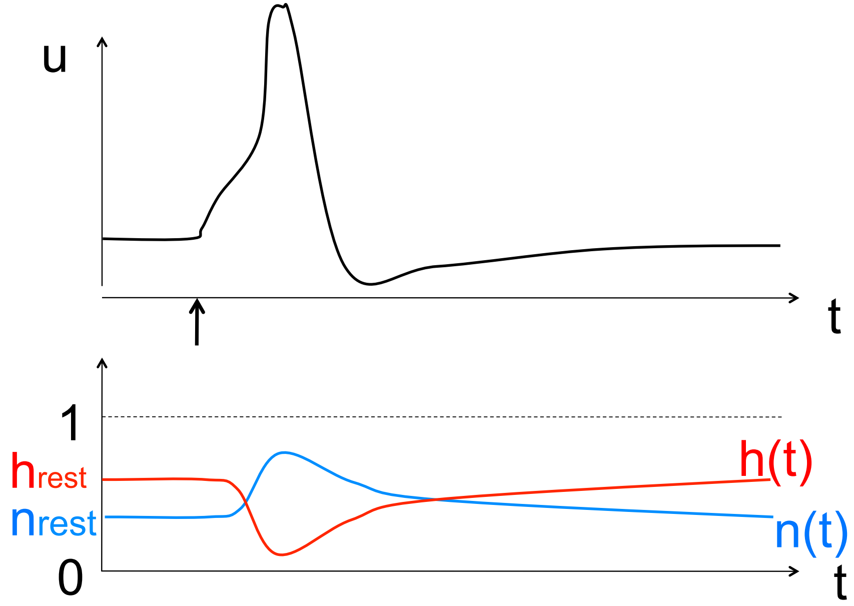
Now !

Neuronal Dynamics – Detour 4.2. Exploit similarities/correlations

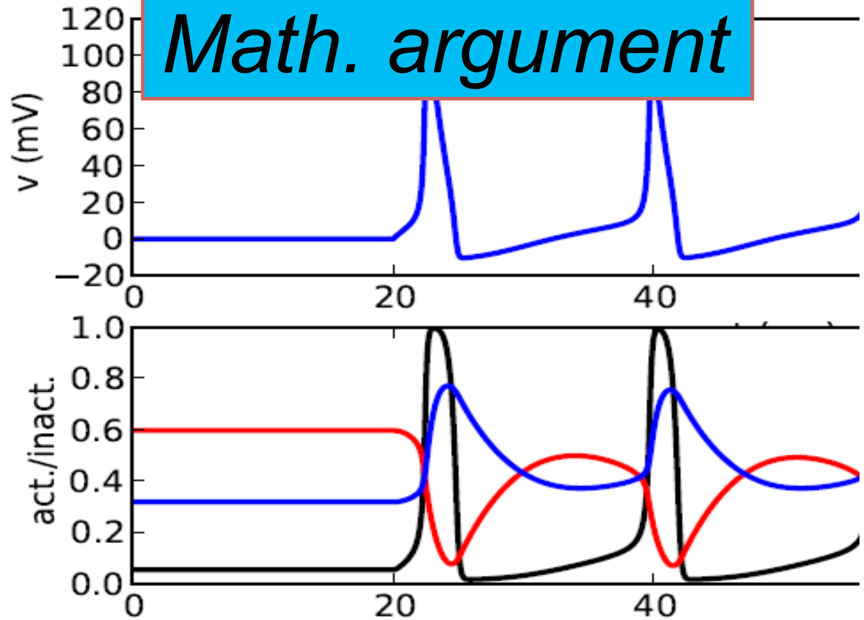
stimulus

$$C\frac{du}{dt} = -g_{Na}m^3h(u-E_{Na}) - g_K n^4(u-E_K) - g_l(u-E_l) + I(t)$$

dynamics of *h* and *n* is similar $\longrightarrow$ $1-h(t) = a\,n(t)$



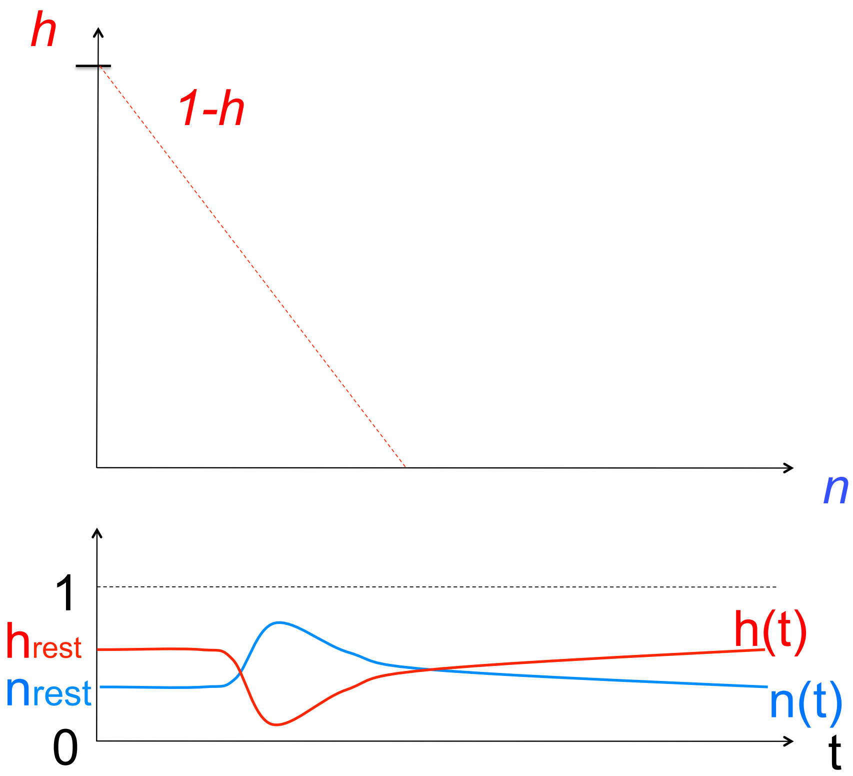
Math. argument

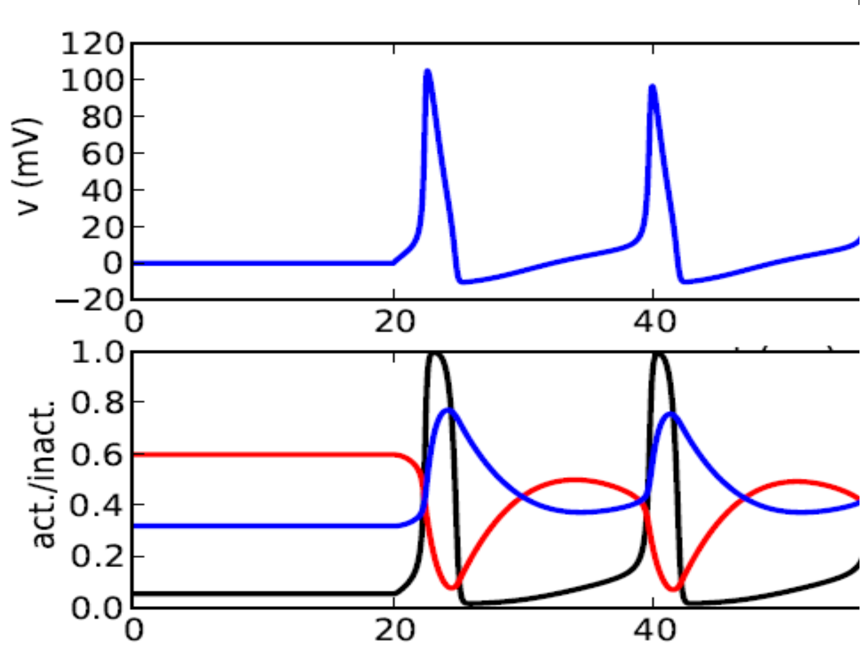


Neuronal Dynamics – Detour 4.2. Exploit similarities/correlations

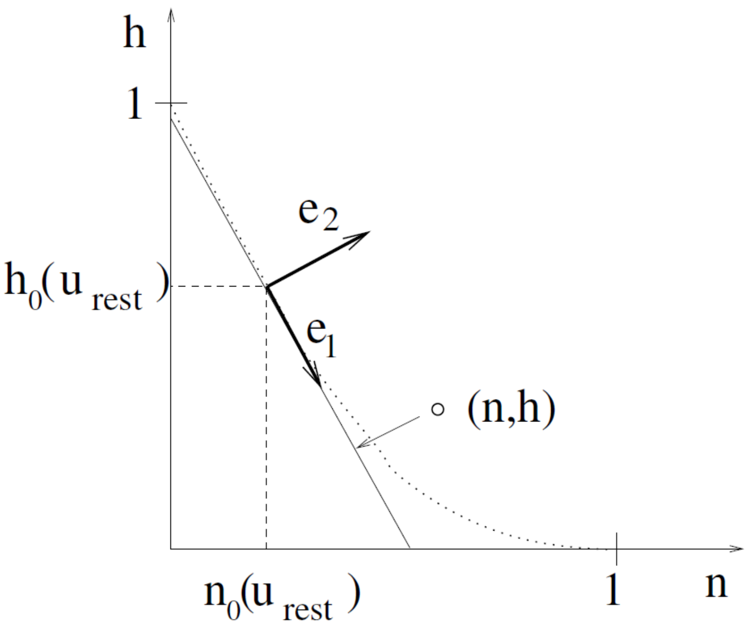
dynamics of *h* and *n* are similar

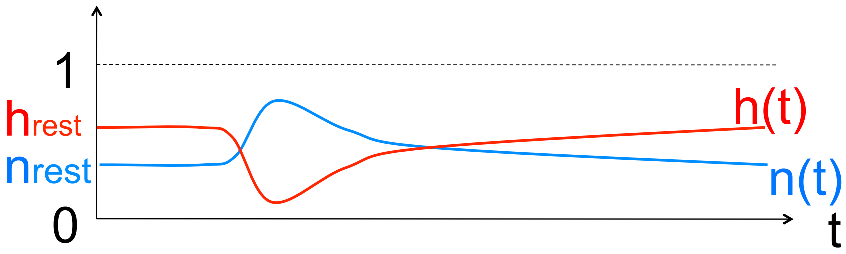
$$1 - h(t) = a\, n(t)$$





Neuronal Dynamics – Detour 4.2. Exploit similarities/correlations





dynamics of *h* and *n* are similar

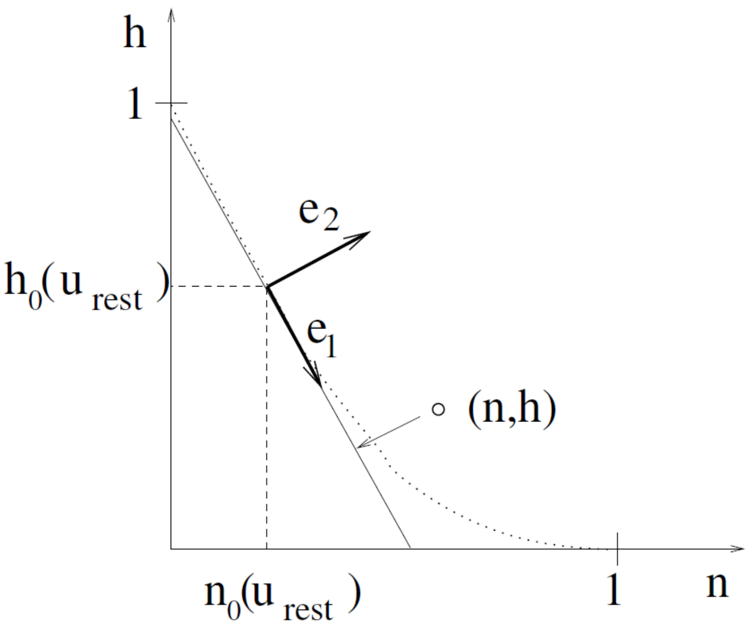
$$1 - h(t) = a\, n(t)$$

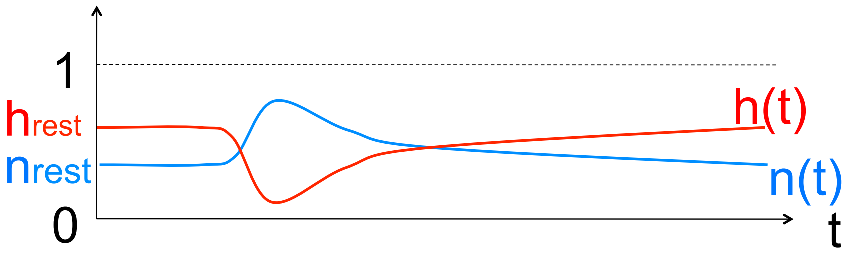
at rest

$$\frac{dh}{dt} = -\frac{h - h_0(u)}{\tau_h(u)}$$

$$\frac{dn}{dt} = -\frac{n - n_0(u)}{\tau_n(u)}$$

Neuronal Dynamics – Detour 4.2. Exploit similarities/correlations





dynamics of *h* and *n* are similar

(i) Rotate coordinate system
(ii) Suppress one coordinate
(iii) Express dynamics in new variable

$$1 - h(t) = a\, n(t) = w(t)$$

$$\frac{dh}{dt} = -\frac{h - h_0(u)}{\tau_h(u)}$$

$$\frac{dn}{dt} = -\frac{n - n_0(u)}{\tau_n(u)}$$

$$\frac{dw}{dt} = -\frac{w - w_0(u)}{\tau_{eff}(u)}$$

Neuronal Dynamics – 4.1. Reduction of Hodgkin-Huxley model

$$C\frac{du}{dt} = \underbrace{-g_{Na}[m(t)]^3 h(t)(u(t)-E_{Na})}_{I_{Na}} \underbrace{-g_K[n(t)]^4(u(t)-E_K)}_{I_K} \underbrace{-g_l(u(t)-E_l)}_{I_{leak}} + I(t)$$

$$C\frac{du}{dt} = -g_{Na}\, m_0(u)^3 (1-w)(u-E_{Na}) - g_K[\frac{w}{a}]^4(u-E_K) - g_l(u-E_l) + I(t)$$

1) dynamics of *m* are fast → $m(t) = m_0(u(t))$

2) dynamics of *h* and *n* are similar → $\underbrace{1-h(t)}_{w(t)} = \underbrace{a\,n(t)}_{w(t)}$

$$\frac{dh}{dt} = -\frac{h-h_0(u)}{\tau_h(u)}$$

$$\frac{dn}{dt} = -\frac{n-n_0(u)}{\tau_n(u)}$$

→

$$\frac{dw}{dt} = -\frac{w-w_0(u)}{\tau_{eff}(u)}$$

Neuronal Dynamics – 4.1. Reduction of Hodgkin-Huxley model

$$C\frac{du}{dt} = \overbrace{-g_{Na}\, m_0(u)^3 (1-w)(u-E_{Na})}^{I_{Na}} \overbrace{- g_K \left(\frac{w}{a}\right)^4 (u-E_K)}^{I_K} \overbrace{- g_l (u-E_l)}^{I_{leak}} + I(t)$$

$$\frac{dw}{dt} = -\frac{w - w_0(u)}{\tau_{eff}(u)}$$

$$\tau \frac{du}{dt} = F(u(t), w(t)) + R\,I(t)$$

$$\tau_w \frac{dw}{dt} = G(u(t), w(t))$$

Neuronal Dynamics – 4.1. Reduction to 2 dimensions

2-dimensional equation

$$\tau \frac{du}{dt} = F(u,w) + R\,I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

Enables graphical analysis!

-Discussion of threshold
-Repetitive firing
-Type I and II

Neuronal Dynamics – Quiz 4.3.

Exploiting similarities:

A sufficient condition to replace two gating variables *r,s* by a single gating variable *w* is

[ ] Both *r* and *s* have the same time constant (as a function of u)

[ ] Both *r* and *s* have the same activation function

[ ] Both *r* and *s* have the same time constant (as a function of u) AND the same activation function

[ ] Both *r* and *s* have the same time constant (as a function of u) AND activation functions that are identical after some additive rescaling

[ ] Both *r* and *s* have the same time constant (as a function of u) AND activation functions that are identical after some multiplicative rescaling