Week 6 – part 4 : Comparison of noise models

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Neuronal Dynamics:
Computational Neuroscience of Single Neurons

Week 6 – Noise models:
Escape noise

Wulfram Gerstner

EPFL, Lausanne, Switzerland

√ **6.1 Escape noise**
- stochastic intensity and point process

√ **6.2 Interspike interval distribution**
- Time-dependend renewal process
- Firing probability in discrete time

√ **6.3 Likelihood of a spike train**
- generative model

6.4 Comparison of noise models
- escape noise vs. diffusive noise
- from diffusive noise to escape noise

6.5. Rate code vs. Temporal Code
- timing codes
- stochastic resonance

Week 6 – part 4 : Comparison of noise models

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

√ **6.1 Escape noise**
- stochastic intensity and point process

√ **6.2 Interspike interval distribution**
- Time-dependend renewal process
- Firing probability in discrete time

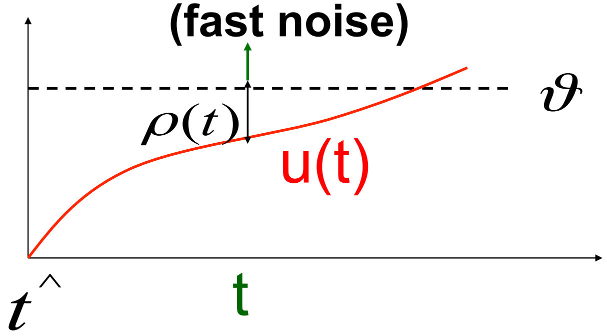
√ **6.3 Likelihood of a spike train**
- generative model

6.4 Comparison of noise models
- escape noise vs. diffusive noise
- from diffusive noise to escape noise

6.5. Rate code vs. Temporal Code
- timing codes
- stochastic resonance

Neuronal Dynamics – 6.4. Comparison of Noise Models

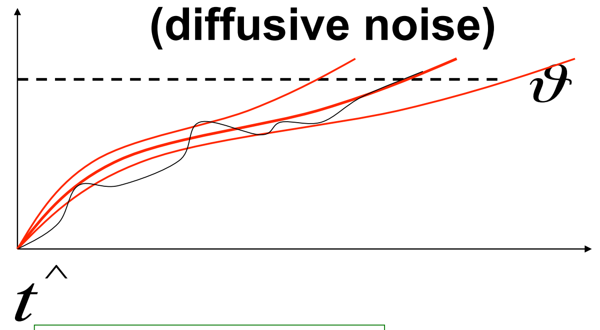
escape process
(fast noise)



escape rate

$$\rho(t) = f(u(t) - \vartheta)$$

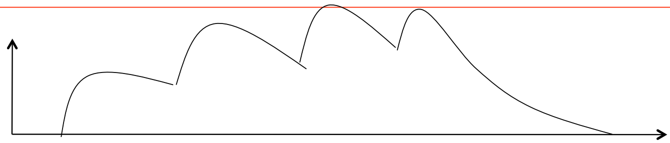
stochastic spike arrival
(diffusive noise)



noisy integration

$$\tau \cdot \frac{du_i}{dt} = -u_i + RI + \xi(t)$$

Poisson spike arrival: **Mean and autocorrelation of filtered signal**



$$S(t) = \sum_f \delta(t - t^f)$$

$$F(s)$$

Filter

$$x(t) = \int F(s) S(t-s) ds$$

Assumption:
stochastic spiking
rate $\nu(t)$

$$\langle x(t) \rangle = \int F(s) \langle S(t-s) \rangle ds$$

mean $$\langle x(t) \rangle = \int F(s) \langle \nu(t-s) \rangle ds$$

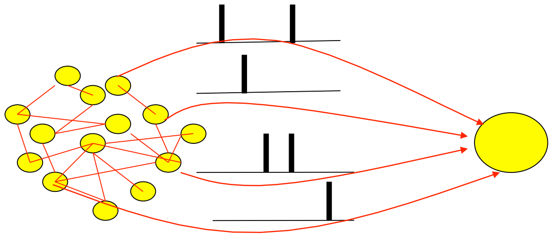
Autocorrelation of output

$$\langle x(t) x(t') \rangle = \left\langle \int F(s) S(t-s) ds \int F(s') S(t'-s') ds' \right\rangle$$

$$\langle x(t) x(t') \rangle = \int F(s) F(s') \underline{\langle S(t-s) S(t'-s') \rangle} ds ds'$$

Autocorrelation of input

Diffusive noise (stochastic spike arrival)



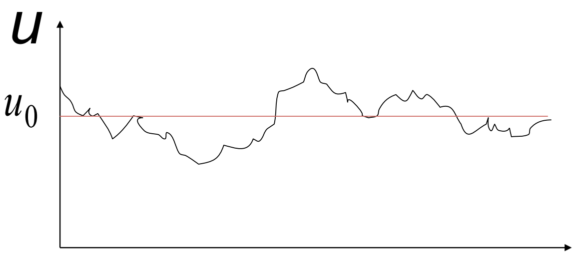
Stochastic spike arrival:
excitation, total rate R_e
inhibition, total rate R_i

Synaptic current pulses

$$\tau \frac{d}{dt} u = -(u - u_{rest}) + \underbrace{\sum_{k,f} \frac{q_e}{C} \delta(t - t_k^f)}_{\text{EPSC}} - \underbrace{\sum_{k',f'} \frac{q_i}{C} \delta(t - t_{k'}^{f'})}_{\text{IPSC}}$$

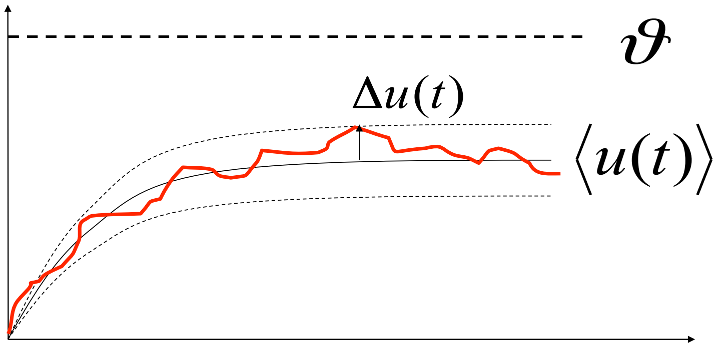
$$\tau \frac{d}{dt} u = -(u - u_{rest}) + R\,I(t) + \xi(t)$$

Blackboard

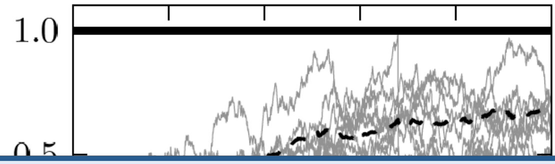


Langevin equation,
Ornstein Uhlenbeck process

Diffusive noise (stochastic spike arrival)



$$\tau \frac{d}{dt} u = -(u - u_{rest}) \quad + R\,I(t) + \quad \xi(t)$$

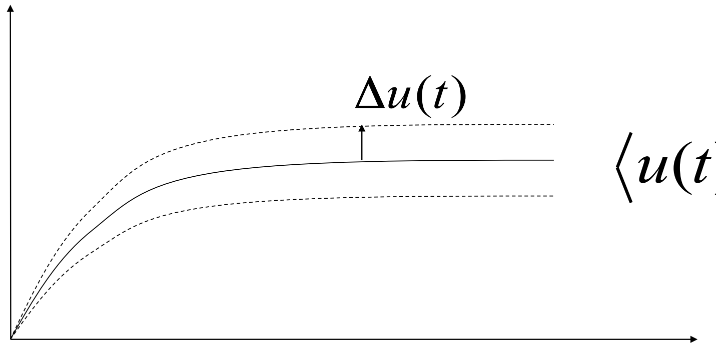


$$\langle \Delta u(t) \Delta u(t) \rangle = \langle u(t) u(t) \rangle - \langle u(t) \rangle^2 =$$

$$\langle \Delta u(t') \Delta u(t) \rangle = \langle u(t) u(t') \rangle - \langle u(t) \rangle \langle u(t') \rangle =$$

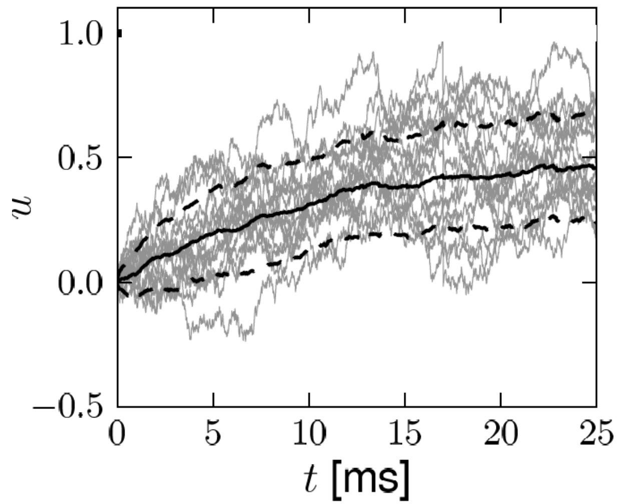
Math argument:
- *no threshold*
- *trajectory starts at known value*

Diffusive noise (stochastic spike arrival)



$$\langle u(t) \rangle = u_0(t)$$

$$\tau \frac{d}{dt} u = -(u - u_{rest}) \quad + RI(t) + \quad \xi(t)$$



$$\langle \Delta u(t) \Delta u(t) \rangle = \langle u(t) u(t) \rangle - \langle u(t) \rangle^2 =$$

$$\langle \Delta u(t') \Delta u(t) \rangle = \langle u(t) u(t') \rangle - \langle u(t) \rangle \langle u(t') \rangle =$$

Math argument

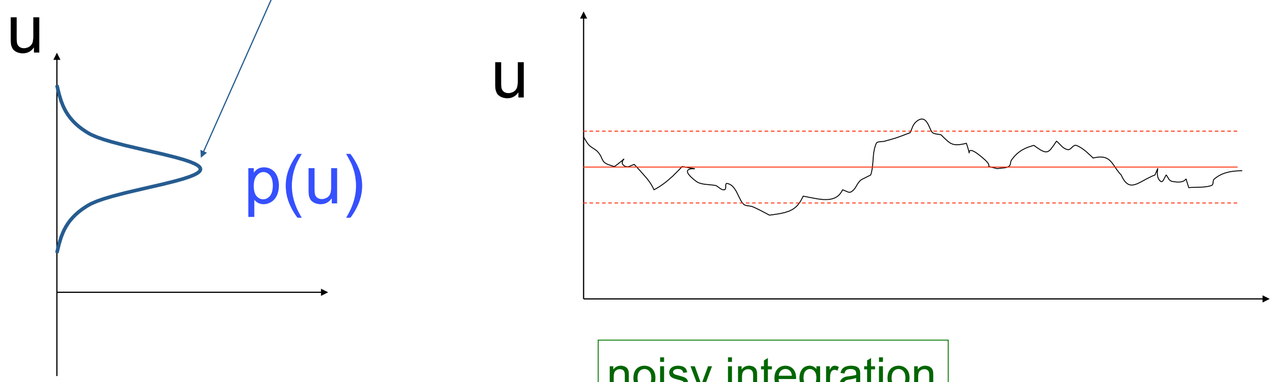
$$p(u,t) = \frac{1}{\sqrt{2\pi \langle \Delta u^2(t) \rangle}} \exp\left\{ -\frac{[u(t|\hat{t}) - u_0(t)]^2}{2 \langle \Delta u^2(t) \rangle} \right\}$$

$$\langle [\Delta u(t)]^2 \rangle = \sigma_u^2 [1 - \exp(-2t/\tau)]$$

Neuronal Dynamics – 6.4. Diffusive noise/stoch. arrival

A) No threshold, stationary input

Membrane potential density: Gaussian



constant input rates
no threshold

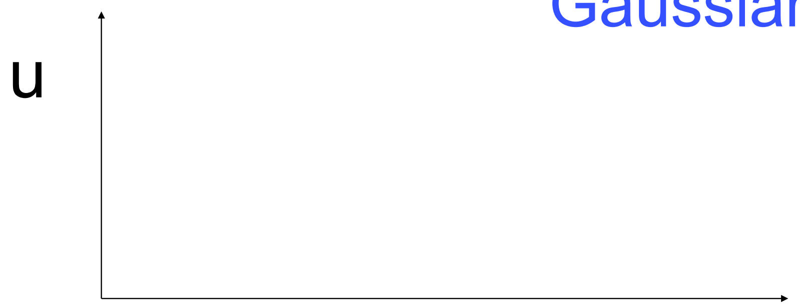
noisy integration

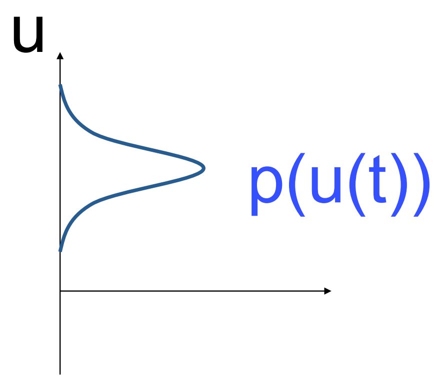
$$\tau \cdot \frac{du_i}{dt} = -u_i + RI + \xi(t)$$

Neuronal Dynamics – 6.4. Diffusive noise/stoch. arrival

B) No threshold, oscillatory input

Membrane potential density:
Gaussian at time t





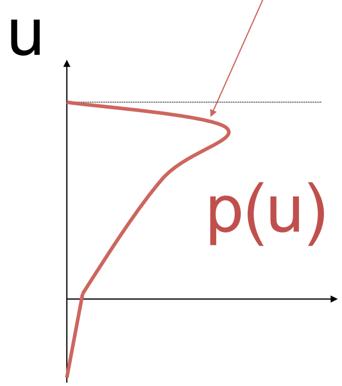
noisy integration

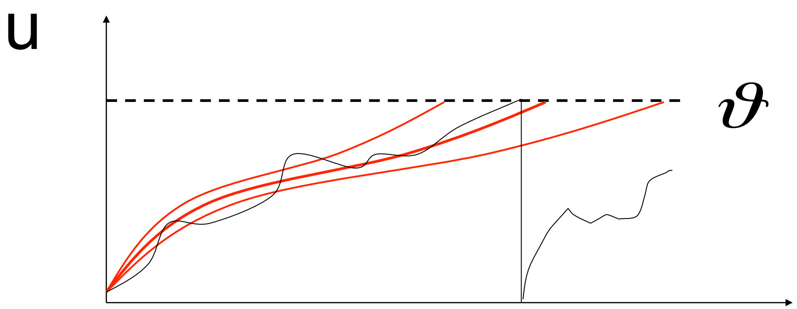
$$\tau \cdot \frac{du_i}{dt} = -u_i + RI + \xi(t)$$

Neuronal Dynamics – 6.4. Diffusive noise/stoch. arrival

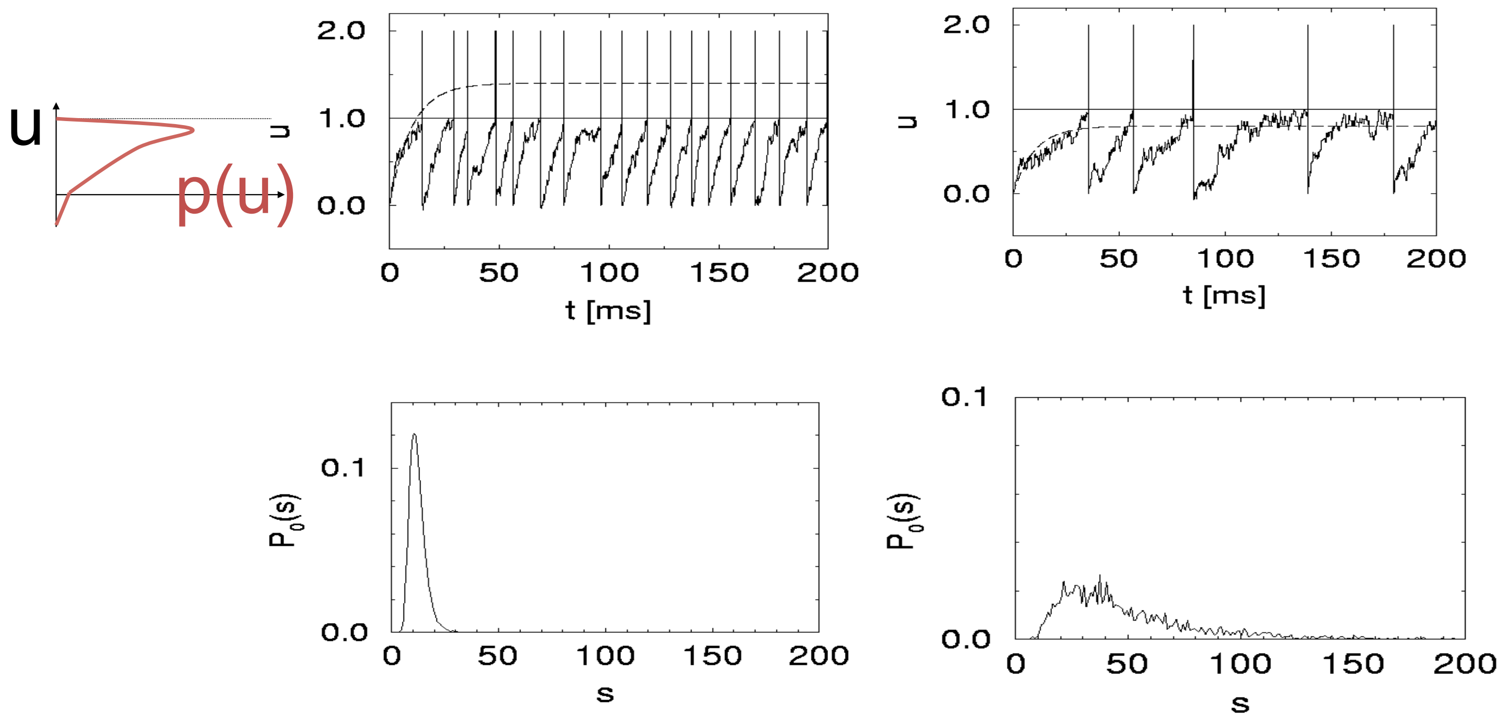
C) With threshold, reset/ stationary input

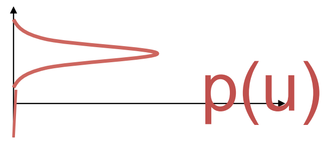
Membrane potential density





Superthreshold vs. Subthreshold regime

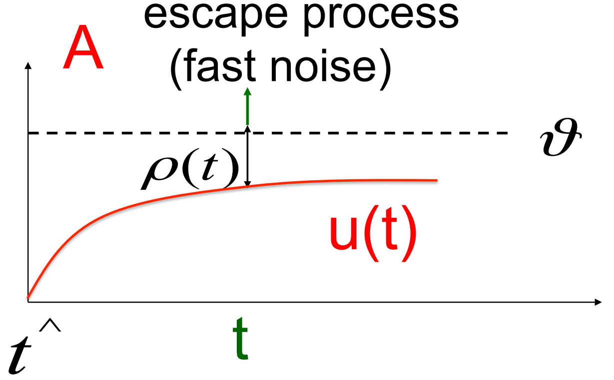




Nearly Gaussian subthreshold distr.

Image:
Gerstner et al. (2013)
Cambridge Univ. Press;
See: Konig et al. (1996)

Neuronal Dynamics – 6.4. Comparison of Noise Models



escape rate

$$\rho(t) = f(u(t) - \vartheta)$$

Interval distribution

$$P_I(t\,|\,\hat{t}) =$$

$$= \rho(t) \cdot \exp\Big(-\underbrace{\int_{\hat{t}}^{t} \rho(t')\,dt'}_{\text{Survivor function}}\Big)$$

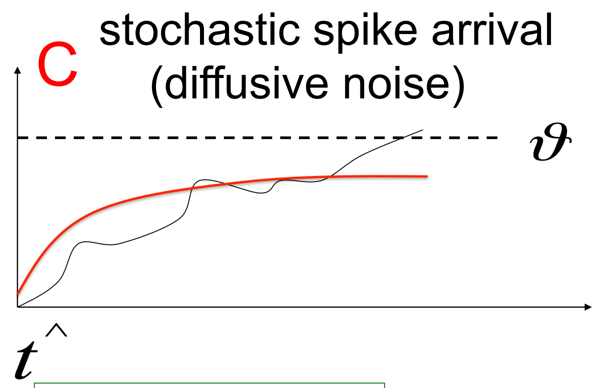
escape rate

Stationary input:
-Mean ISI

$$\langle s \rangle = \tau_m \sqrt{\pi} \int_{\frac{u_r - h_0}{\sigma}}^{\frac{\vartheta - h_0}{\sigma}} du\, \exp\left(u^2\right)\, [1 + \mathrm{erf}(u)]$$

Siegert 1951

-Mean firing rate



noisy integration

$$\tau \cdot \frac{du_i}{dt} = -u_i + RI + \xi(t)$$

noise

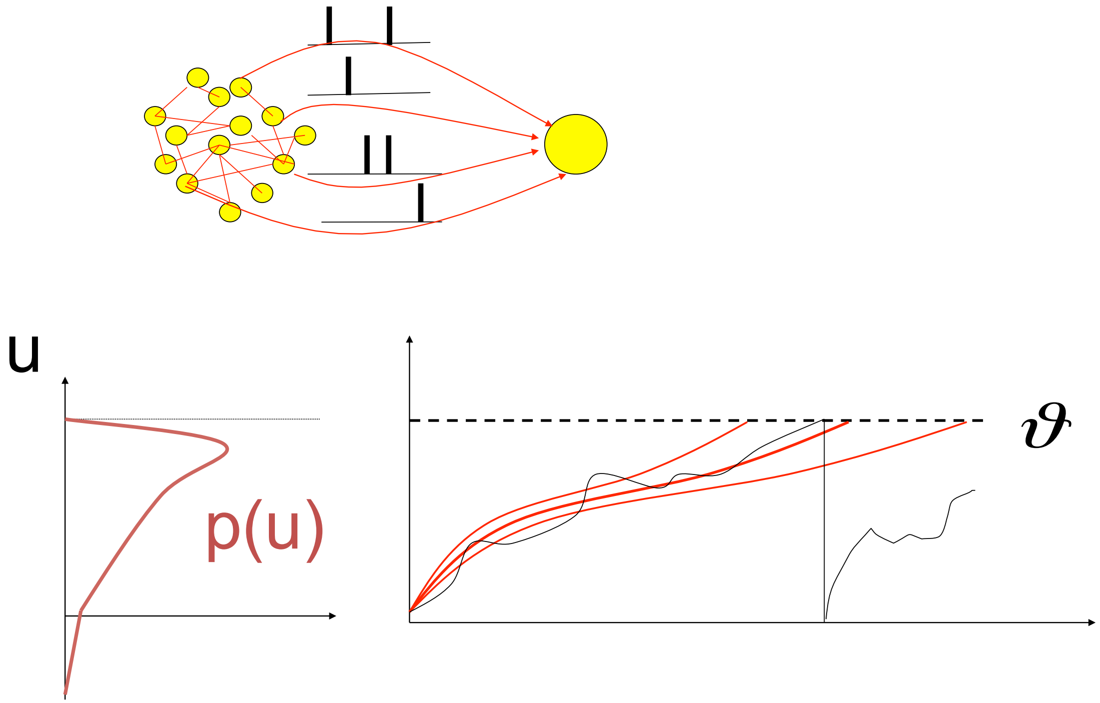
white (fast noise)

synapse (slow noise)

$P_I(t\,|\,\hat{t})$: first passage time problem

(Brunel et al., 2001)

Neuronal Dynamics – 6.4. Comparison of Noise Models



Diffusive noise

- distribution of potential
- mean interspike interval

FOR CONSTANT INPUT

- time dependent-case difficult

Escape noise

- time-dependent interval distribution